Generalizations of string theory

W 5/11 10:50-11:30

Outline

(feel free to add "super-" anywhere; always in superspace)

T-theory

(1) Current algebra: strings ⇒ double field theories

(2) New spaces: field strengths as gauge fields

(3) Chiral strings: boundary cond. ⇒ finite spectrum

- (a) Effective actions: new for (massive) gravity
- (b) Particle amplitudes: scattering eqs. & KLT

(not your father's) F-theory (nor mother's M-theory)

(1) Current algebra: branes ⇒ exceptional gravity

(2) 0th-quantized ghosts

T-theory

Current algebra: strings $\Rightarrow$ double field theories

T-duality (worldsheet, no background) (WS '83)

Fields d=1 dimensionally reduced (Buscher '87)

d>1: O(d, d)/O(d)^2 scalars (Duff '89, Tseytlin '90)

"Double field theory", bosonic & heterotic (WS '93):

- O(D,26)/O(D−1,1)O(25,1); D = 10,26;
  for gauge fields g, B, A (spont. broken O(D,26))
- "strong constraint" (sectioning) $\Leftrightarrow$ (gauge inv.)^2
- "C-bracket" & "D-bracket" (nonabelian for het.)
- $\nabla_A, T_{ABC}, R_{AB}{}^{cd}$, field equations, action
- gauge fixing: LR factorization of Feynman rules

Our recent work (see MH talk for some details)

Spin S_{ab} & dual Σ_{ab} (WS '11, Poláček & WS '13)

- $\oplus$ current super algebra $D_\alpha, P_a, \Omega^\alpha$ (WS '85)
- curvature as torsion
- Lorentz connection couples to string

Type II & RR (Hatsuda, Kamimura, WS '14-'15)

- RR gauge fields in central currents $\Upsilon_{\alpha\alpha'}, \mathsf{F}^{\alpha\alpha'}$
- Type II algebra from string action

3D Type II off-shell background (Poláček & WS '14)

- prepotential in vielbein without derivatives on it
- O(3,3)/O(2,1)^2(= O(2,2))
- O(3,1) M-theory & O(3,2) F-theory (see later)

New spaces: field strengths as gauge fields (WS '11)

T-theory methods generalize to particle field theory, e.g., gravity

- S_{ab}: 1st-quantization of spin
- spin coordinates $\Rightarrow$ curvature as torsion (tangent space as subspace of enlarged manifold)

But also Yang-Mills

- Σ_{ab}: field strength F_{ab} = gauge connection for Σ
- first-order action = $PP\Sigma$ Chern-Simons

Chiral strings: b.c. $\Rightarrow$ finite spectrum (see BZ talk)

Effective actions (Hohm, Zwiebach, WS '13)

- chiral (non-conformal) gauge: $X(z)$ (no $\bar{z}$)
- chiral boundary conditions: $N + \overline{N} = 0$ (not $-$)
- Virasoro algebra $\Rightarrow$ field equations
- complete @ 6 derivatives (α'^2) $\Rightarrow$ ... $+ R^3$
- α' modifications to brackets
- unconstrained fields (not O(26,26))
- cubic action (except for dilaton)
- like 2D CP(n), nonlinear $\sigma \rightarrow$ linear @ O(α'): Lagrange multiplier $\rightarrow$ massive, propagating (understood recently: see below)

Particle amplitudes for all dimensions

"Scattering equations" (Cachazo, He, Yuan '13)

From string-like theories (Mason & Skinner '13)

Derivation from usual string theories (WS '15)

- **chiral boundary conditions (HSZ)**
- **"almost" chiral gauge (HSZ): $\bar{z}$ as IR regulator**
- **usual string vertex operators**

KLT factorization (Huang, Yuan, WS '16)

- **chiral boundary conditions, but conformal gauge**
- **ordinary open strings, except $\eta_{ab} \to -\eta_{ab}$ for $\bar{z}$**
- **sign change cancels 0's against usual poles**
- **fixes earlier bosonic failure: (few) massive modes**

F-theory (& M-theory) (Linch & WS '15)

Before branes: more-than-DFT for exceptional SG (Coimbra, Strickland-Constable, Waldram '11; Berman, Cederwall, Kleinschmidt, Thompson '12)

Current algebra on fundamental branes

- brane background: exceptional (super)gravity
- X: selfdual gauge fields on worldvolume
- worldvolume indices are spacetime indices
- worldvol. metric constrained $\Rightarrow$ nonpropagating
- sectioning for worldvolume (Gauss's law)
- sectioning = 0-modes of dimensional reduction
- solve Virasoro $\rightarrow$ M-theory; Gauss $\rightarrow$ T-theory

0th-quantized ghosts (WS '16)

- $\langle X\, X\rangle = -\ln z^2$ for higher-d z (branes)?
- ghosts for every order of quantization:

(2) usual ghost fields for $g(X), B(X)$, etc.
(2nd-quantization)

(1) worldvolume ghost "fields" c, b for $X(z)$
(1st-quantization)

(0) ghost coordinates for z
(0th-quantization)

Appendix: Some technical details

T-theory current algebra with Ramond-Ramond

Currents: $(S_{ab}, D_\alpha, P_a, \Omega^\alpha, \Sigma_{ab})_\pm\,; \Upsilon_{\alpha\alpha'}, \mathsf{F}^{\alpha\alpha'}$

$$\{D_\pm, D_\pm\} = P_\pm\delta \ , \quad \{D_+, D_-\} = \Upsilon\delta$$

$$[D_\pm, P_\pm] = \Omega_\pm\delta \ , \quad [D_\pm, \mathsf{F}] = \Omega_\mp\delta$$

$$[S_\pm, \textbf{not } \Sigma] \sim \textbf{not } \Sigma \ , \quad [S_\pm, \Sigma_\pm] = \Sigma_\pm\delta + \delta'$$

$$[P_\pm, P_\pm] = \Sigma_\pm\delta + \delta' \ , \quad [\Upsilon, \mathsf{F}] = (\Sigma_+ + \Sigma_-)\delta + \delta'$$

$$\{D_\pm, \Omega_\pm\} = \Sigma_\pm\delta + \delta'$$

1st-order Yang-Mills as Chern-Simons

With torsion $[d_A, d_B\} = T_{AB}{}^C d_C$, the CS form is

$$X_{ABC} = \tfrac{1}{2}A_{[A}d_B A_{C)} - \tfrac{1}{4}A_{[A}T_{BC)}{}^D A_D + \tfrac{1}{3}iA_{[A}A_B A_{C)}$$

In particular, for

$$[p_a, p_b] = \sigma_{ab} \quad \Rightarrow \quad T_{a,b}{}^{cd} = \tfrac{1}{2}\delta^c_{[a}\delta^d_{b]}$$

we choose

$$L = \tfrac{1}{2}a^{a,b,cd}X_{a,b,cd}\ , \quad a^{a,b,cd} = \eta^{a[c}\eta^{d]b}$$

Then, labeling $A_p \to A, A_\sigma \to F$,

$$L \sim A\sigma A + FpA - FF + FAA$$

gives the usual 1st-order action after $\sigma \to 0$.

Scattering equation δ from almost-chiral gauge

Change in boundary condition,

$$\Delta = -\ln z - \ln \bar{z} \;\to\; -\ln z + \ln \bar{z}$$

followed by change in gauge (with $\beta \to \infty$),

$$\to \quad -\ln z + \ln(\bar{z} - \beta z) \;\to\; -\frac{1}{\beta}\frac{\bar{z}}{z}$$

modifies $\bar{z}$ half of Koba-Nielsen integral

$$\int d\bar{z}_i \, \exp \frac{1}{2\beta} \sum k_i \cdot k_j \frac{\bar{z}_{ij}}{z_{ij}} \sim \prod_i \delta \left(\sum_j \frac{k_i \cdot k_j}{z_{ij}} \right)$$

KLT for chiral string (toy example, $m=0$)

Normal $\rightarrow$ chiral [$\sin(\pi t) = -\pi/\Gamma(-t)\Gamma(1+t)$]:

$$-\frac{\Gamma(-s)\Gamma(-t)}{\Gamma(u)}\frac{1}{\Gamma(-t)\Gamma(1+t)}\frac{\Gamma(1-t)\Gamma(-u)}{\Gamma(1+s)}$$

$$=\frac{1}{s}\frac{\Gamma(-s)\Gamma(-t)\Gamma(-u)}{\Gamma(s)\Gamma(t)\Gamma(u)}$$

$$\rightarrow\ -\frac{\Gamma(-s)\Gamma(-t)}{\Gamma(u)}\frac{1}{\Gamma(-t)\Gamma(1+t)}\frac{\Gamma(1+t)\Gamma(+u)}{\Gamma(1-s)}$$

$$=\frac{1}{s}$$

F-theory dimensional reduction & sectioning

$$\sigma - \text{Virasoro} \qquad \mathcal{S} \equiv PP$$

$$\text{dim. reduction} \qquad \overset{\circ}{\mathcal{S}} \equiv pP \qquad \mathcal{U} \equiv \partial P$$

$$\text{sectioning} \qquad \underset{\circ}{\mathcal{S}} \equiv pp \qquad \underset{\circ}{\mathcal{U}} \equiv \partial p \qquad \mathcal{V} \equiv \partial\partial$$

P = bosonic current (selfdual field strength);
p = 0-mode, ∂ for worldvolume

Downward: $P \to p$ (otherwise same)

Dimensional reduction kills X's ($\mathcal{U}$ = Gauss)

Section conditions kill 0-modes (background) & σ's

$(\overset{\circ}{\mathcal{S}}, \underset{\circ}{\mathcal{S}}) = 0 \Rightarrow$ M-theory; $(\mathcal{U}, \underset{\circ}{\mathcal{U}}, \mathcal{V}) = 0 \Rightarrow$ T-theory

A little F-theory current algebra

d worldvolume coordinates: $\tau, \sigma_{\mathcal{M}}$

Spacetime coordinates: $\Theta^{\mu}, X^{m}, Y^{m'}, \widetilde{Y}_{\mathcal{M}m'}$

(32 Θ**'s,** $<$**10 worldvolume scalars** Y**, their duals** $\widetilde{Y}$**)**

Currents: $(S,)D_{\mu}, P_{m}, \Upsilon_{m'}, \mathsf{F}^{\mathcal{M}m'}, \Omega^{\mathcal{M}\mu}(, \Sigma)$

$$[\triangleright_{M}, \triangleright_{N}\} = f_{MN}{}^{P} \triangleright_{P}\delta + 2i\eta_{MN\mathcal{R}}\partial^{\mathcal{R}}\delta$$

$$\Rightarrow \quad f_{[MN|}{}^{Q} f_{Q|P)}{}^{R} = 0 = f_{M(N|}{}^{Q}\eta_{Q|P]\mathcal{R}}$$

$$\mathcal{S}^{\mathcal{R}} = \eta^{MN\mathcal{R}} \triangleright_{N} \triangleright_{M}$$

f**'s** $\sim$ **super YM, but doubled spinor indices**

Types of F-theories (N=1 D-dim. SG bosons in G/H)

D	d	H	G	X	σ
1	3	GL(1)	GL(2)	2+1	2
2	4	GL(2)	SL(3)SL(2)	(3,2)	(3′,1)
3	6	Sp(4)	SL(5)	10	5′
4	11	Sp(4,C)	SO(5,5)	16	10
5	28	USp(4,4)	$E_{6(6)}$	27	27′
6	134	SU*(8)	$E_{7(7)}$	56	133

G = "E_{D+1}",
H = covering of SO(D−1,1) with double argument,
H′ = SO(10−D)
Y = vector of H′, Θ = spinor of H⊗H′ (32)