Outline lectures van Nieuwenhuizen.

In this set of 10 lectures we shall begin by explaining what gravitational and gauge anomalies are. Then we discuss the cancellation of anomalies (both gravitational and gauge anomalies) in the ten-dimensional susy and sugra field theories which describe the massless spectrum of superstring theory. The infinities due to one-loop contributions in open **string** theory cancel, hence the corresponding field theories (with some massless and infinitely many massive fields) should also be finite. A finite theory cannot have anomalies, and massive fields do not produce anomalies, hence the anomalies in the massless susy and sugra theories **must** cancel. By direct calculation, Alvarez-Gaumé and Witten found that they indeed cancel in II B sugra. But in $N=1$ sugra and $N=1$ SYM theory (taking each separately, or coupling them), the anomalies did not seem to cancel. Green and Schwarz found the solution to this paradox: one needs to add to the action finite gauge-violating counterterms ΔS, whose gauge variation cancels the anomalies. The need for counterterms to cancel spurious anomalies was well-known to field theorists but the counterterms for the **susy** theory needed the **sugra** field $B_{\mu\nu}=-B_{\nu\mu}$. This was unexpected for field theorists, but explained by string theory.

The actual construction of the counterterms ΔS uses interesting but advanced concepts:

- the notion of covariant and consistent anomalies.
- the descent equations which produce consistent anomalies from covariant anomalies.
- Chern-Simons terms as building blocks of ΔS.
- group theory to write traces $\mathrm{Tr}\, F^6$ of products of Yang-Mills curvatures in the adjoint representation as sums of products of traces $\mathrm{tr}\, F^m$ in the defining representations of orthogonal groups $SO(N)$.
- more group theory to prove that for the exceptional group E_8 one has $\mathrm{Tr}\, F^6 = \frac{1}{7200}$ $(\mathrm{Tr}\, F^2)^2$.

The central role of Chern-Simons terms in sugra was first found in the Maxwell-Einstein sugra theory (with abelian Chern-Simons terms), and later extended to the Yang-Mills-Einstein sugra theory (with nonabelian Chern-Simons terms). In the actions for these theories one finds the expression $\mathcal{L}_B=(H_{\mu\nu\rho}-\omega_{\mu\nu\rho,Y})^2$, where $H_{\mu\nu\rho}$ is the curl of $B_{\mu\nu}$ and $\omega_{\mu\nu\rho,Y}$ is the CS term for a (non)abelian gauge group. Cancellation of gauge **and** gravitational anomalies requires also a Lorentz Chern-Simons term $\omega_{\mu\nu\rho,L}$ in $\mathcal{L}_B$. Cancellation of pure gravitational anomalies required a group with 496 generators, and string theory could provide one: adding Chan-Paton factors to the ends of the open string, one can obtain the group $SO(32)$ with 496 generators. However, a second solution was found which also canceled all gauge (YM) and gravitational (Lorentz) anomalies: the group $E_8\times E_8$ which also has 496 generators. This second solution was produced by field theory (anomaly cancellation in $N=1$ sugra coupled to $N=1$ SYM), but no string theory was known with this gauge group. Sometimes string theory provides solutions to problems in field theory (such as the prediction of anomaly cancellation mentioned above), but at other times field theory (sugra) offers solutions to problems in string theory. The case of the $E_8 \times E_8$ string is an example of the latter case. (Other examples of the latter case are the GSO projection operators, and the existence of M theory in eleven dimensions.)

The new string theory which produces the gauge group $E_8 \times E_8$ is the heterotic string, with the $d = 10$ spinning string in the left-moving sector, and the $d = 26$ bosonic string in the right moving sector. The 16 extra coordinates, denoted by ϕ^I, are real chiral bosons (the same as selfdual bosons) in two Minkowski dimensions. Much has been written about all the problems that arise if one tries to construct an action for a chiral boson, but we shall bypass these discussions, and simply consider nonchiral bosons with only the ground state in the left-moving sector, but no excited states. This effectively produces chiral bosons. The extra 16 dimensions are compactified to a torus, and its covering space must be an even selfdual lattice. There are in 16 dimensions two even selfdual lattices: one generated by the roots of $E_8 \times E_8$, and the other generated by the roots and one of the spinor representations of $SO(32)$. There is an equivalent formulation of the heterotic string with 32 extra fermions instead of 16 extra bosons; this formulation uses only concepts of the NSR string which we are already familiar with, so we will start with this fermionic formulation. From a technical point of view, the most interesting part of our discussion of the heterotic string is the theory of lattices, and the construction of vertex operators for simply-laced Lie algebras. For this we shall need some group theory for $SO(2N)$ groups ($SO(32)$, but also $SO(16)$ which is a subgroup of E_8). We shall not assume this knowledge but provide it.

The last part of the lectures deals with the Kaluza-Klein reduction of II B supergravity from 10 to 5 dimensions, by compactifying part of space to a 5-sphere S_5. This entails the construction of "spherical harmonics" for all fields with spins $0, \frac{1}{2}, 1, \frac{3}{2}$ and 2, and leads to a spectrum of massless and massive states which forms the basis for the AdS/CFT correspondence, which will be discussed later.