Is F-theory a theory?

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Geometrical Aspects of Supersymmetry

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What the F is “F-theory”?

We still don’t even know what the “M” in “M-theory” is.

String theory > just supergravity (low energy).

Supergravity > just compactifications (vacua).

“Stringy” theory with all symmetries (S,T,U,...) manifest?*

- String & M theories ← “semi-unitary gauges” with spontaneous symmetry breaking by solving worldvolume constraints (cf. lightcone).
- More symmetry manifest ← “covariant gauges” with ghost worldvolume coordinates.

*Vafa

The story so far:

- consistently quantizable branes
- Spacetime coordinates $X(\sigma)$ = worldvolume gauge field with selfdual field strength.
- $\Rightarrow$ Spacetime indices = worldvolume indices.
- Zeroth-quantization: σ also = gauge field?

Want to better understand...

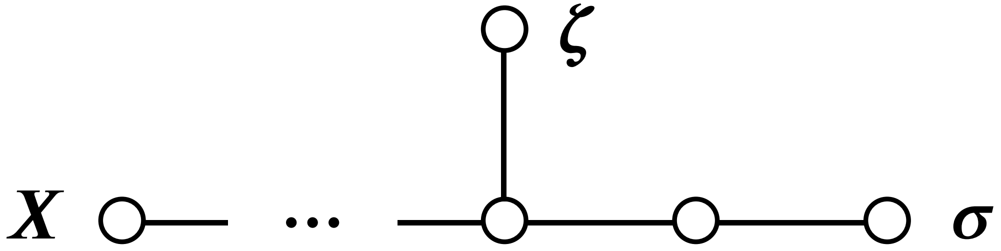
- supersymmetry (super σ)
- worldvolume Lagrangian (worldvolume metric)
- (STU) covariant quantization (ghost σ)
- higher dimensions (E_{11}?*)

*West

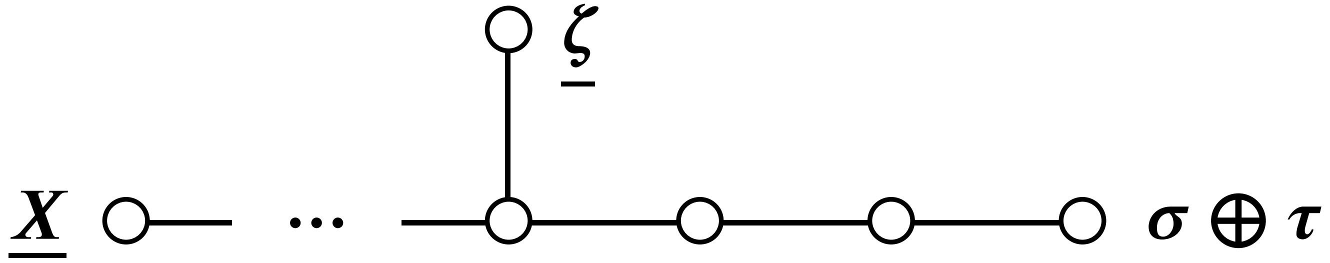
Symmetries

	exceptional		tangent	∪:	d⊃d−1
ℒ(agrangian)	F	⊃	L	⊃:	maximal
	∪		∪		compact
ℋ(amiltonian)	G	⊃	H		subgroup

G



F



Some cases

D	d	F	G	L	H
0	2	GL(2)	GL(1)	GL(1,C)	I
1	3	GL(3)	GL(2)	GL(2)	SO(1,1)
2	4	SL(4)SL(2)	SL(3)SL(2)	$GL(2)^2$	GL(2)
3	6	SL(6)	SL(5)	GL(4)	Sp(4)
4	12	SO(6,6)	SO(5,5)	GL(4,C)	Sp(4,C)
5	56	$E_{7(7)}$	$E_{6(6)}$	U*(8)	USp(4,4)
6	∞?	∞?	$E_{7(7)}$	$U^*(8)^2$	SU*(8)
7	∞?	∞?	$E_{8(8)}$	U*(16)	SO*(16)

- D = spacetime, after reduction to usual string theory
- d = worldvolume, before reduction

H = Lorentz covering with double argument

For lower D, resemble gravity/“double field theory”:

D	F	G	L	H
0-3	GL(d)	GL(d$-$1)	SO(d)	SO(d$-$1)
4	SO($\frac{d}{2}$,$\frac{d}{2}$)	SO($\frac{d}{2}$$-$1,$\frac{d}{2}$$-$1)	SO($\frac{d}{2}$,C)	SO($\frac{d}{2}$$-$1,C)

F groups in D dimensions are Wick rotations of N = D+2 exceptional symmetries for 4D sugra:

D	F_D	E_N	N
0	GL(2)	U(2)	2
1	GL(3)	U(3)	3
2	SL(4)SL(2)	SU(4)SU(1,1)	4
3	SL(6)	SU(5,1)	5
4	SO(6,6)	SO*(12)	6
5	$E_{7(7)}$	$E_{7(7)}$	7 (8)

Worldvolume fields (Lagrangian)

$$\delta X = \partial\zeta\,, \quad F = \partial X$$

D	F	pattern	σ	ζ	X	F
0	GL(2)	forms	2	0	1	2
1	GL(3)	forms	3	$0 \oplus 1$	$1 \oplus 3$	$3 \oplus 3'$
2	SL(4)SL(2)	forms	$(4,1)$	$(1,2)$	$(4,2)$	$(6,2)$
3	SL(6)	forms	6	6	15	20
4	SO(6,6)	spinors	12	32	$32'$	32
5	$\mathrm{E}_{7(7)}$	infinite	56	912	133	56

Imposing selfduality on F breaks symmetry F → L (in absence of background).

Current algebra (Hamiltonian)

$\mathscr{L} \to \mathscr{H}$: $X \to X_\tau \oplus X_\sigma$

- X_τ = Lagrange multiplier for Gauss (cf. part of ζ)
- X_σ = “physical” (cf. selfdual part $\triangleright$ of F)

$$i[\triangleright_A, \triangleright_B\} = \delta f_{AB}{}^C \triangleright_C - 2\eta_{AB}{}^c \mathscr{D}_c \delta$$

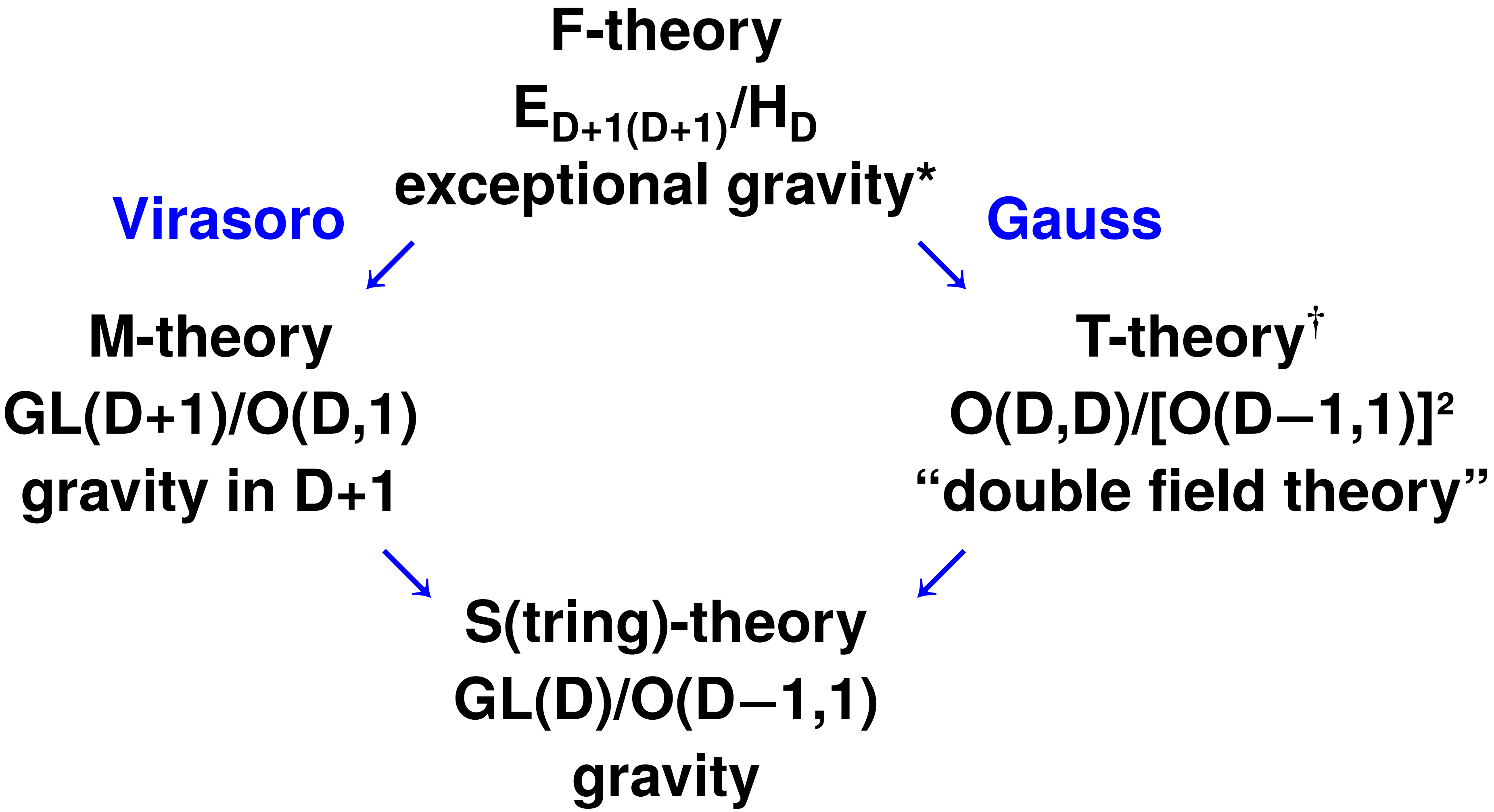
$$\mathscr{D}_a = g_a{}^m \partial_m \, , \quad \triangleright_A = R_A{}^M P_M + (\eta_{AB}{}^c + B_{AB}{}^c)(\mathscr{D}_c X^M) R_M{}^B$$

$$\begin{cases} \text{Virasoro:} \; \mathscr{S}_a = \eta^{BC}{}_a \triangleright_B \triangleright_C \\ \text{Gauss:} \quad \mathscr{U}_{\mathscr{A}} = \mathscr{f}^{bC}{}_{\mathscr{A}} \mathscr{D}_b \triangleright_C \end{cases}$$

Background: $(\mathscr{D}_a, \triangleright_A, f_{AB}{}^C) \to (E_a{}^m \mathscr{D}_m, E_A{}^M \triangleright_M, T_{AB}{}^C)$

Gauge transformations: $\int \Lambda^M \triangleright_M$; $(E_a{}^m, E_A{}^M) \in$ G/H

Reductions (& extended gravities G/H)



F-theory

$E_{D+1(D+1)}/H_D$

exceptional gravity[*]

Virasoro

Gauss

M-theory

GL(D+1)/O(D,1)

gravity in D+1

T-theory[†]

$O(D,D)/[O(D-1,1)]^2$

“double field theory”

S(tring)-theory

$GL(D)/O(D-1,1)$

gravity

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[*] Coimbra, Strickland-Constable, & Waldram; Berman, Cederwall, Kleinschmidt, & Thompson

[†] WS, but string has Duff/Tseytlin doubled X

Simple example: 3D bosonic (d= 6)

$$\begin{tikzcd}
 & \begin{array}{c}\mathbf{F}\\ \mathbf{SL(5)/O(3,2)}\end{array} \arrow[dl] \arrow[dr] & \\
\begin{array}{c}\mathbf{M}\\ \mathbf{GL(4)/O(3,1)}\end{array} \arrow[dr] & & \begin{array}{c}\mathbf{T}\\ \mathbf{SL(4)/O(2,1)^2}\end{array} \arrow[dl] \\
 & \begin{array}{c}\mathbf{S}\\ \mathbf{GL(3)/O(2,1)}\end{array} &
\end{tikzcd}$$

σ_m = 5, $X^{[mn]}$ = 10

$O(2,1)^2 = O(2,2)$

Defining-rep G-symmetry indices:
time = –1, 0; space = 1,2,3

F: m = –1,0,1,2,3
M: i = 0,1,2,3
T: i = –1,0,1,2
S: ι = 0,1,2

Sectioning, in $\mathscr{H}$ formalism, uses $P \rightarrow p$:

$$\left\{\begin{array}{llll} \text{Virasoro: } \varepsilon^{mnpqr}P_{np}P_{qr} & \rightarrow & pP & \rightarrow pp \\ \text{Gauss:} & & \partial^n P_{nm} & \rightarrow \partial p \end{array}\right.$$

Apply also to products of fields
⇒ the 2 derivatives in constraint to 2 different fields.

- Virasoro: F → M: $P_{ij} = p_{ij} = 0$,
  leaves σ_m & X^{-1i} (still 5-brane, but in D = 4)
- Gauss: F → T: $P_{3i} = p_{3i} = \partial^i = 0$,
  leaves σ_3 & X^{ij} (string, but in D = 3+3, SD & $\overline{\text{SD}}$ X^{ij})
- both: F → S: leaves σ_3 & $X^{-1\imath}$ (string in D = 3)

P.S. Martin Roček is a great guy!