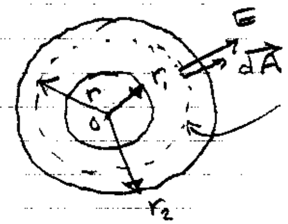
HW #3

22-21



spherical shells $\begin{cases} \text{charge density } \sigma_1 & (r_1) \\ \quad '' \quad \sigma_2 & (r_2) \end{cases}$

<u>Choose Gaussian surface:</u>

For any sphere centered at O with any radius r, E field is always in radial direction. This means $\vec{E}\cdot\vec{dA} = EdA\cos 0° = EdA$. This says flux for such a sphere

is: $\Phi_{\substack{\text{sphere}\\ \text{at } r}} = (\text{Area of sphere})\, E(r)$
$= 4\pi R^2 E(r)$

Gauss's Law: $\Phi = \dfrac{Q_{encl.}}{\epsilon_0}$

So $4\pi\epsilon_0 E(r) = \underline{Q_{\text{inside } r}}$

or $E(r) = \dfrac{Q_{\text{inside}}}{4\pi r^2 \epsilon_0}$

a) $r < r_1$; $Q_{\text{inside } r_1} = 0 \rightarrow E_{(r<r_1)} = 0$

b) $r_1 < r < r_2$; $Q_{r_1<r<r_2} = (4\pi r_1^2 \sigma_1)$

$\rightarrow E(r)_{r_1<r<r_2} = \dfrac{r_1^2\sigma_1}{r^2\epsilon_0}$

c) $r_2 < r$; $Q_{r_2<r} = 4\pi(r_1^2\sigma_1 + r_2^2\sigma_2)$

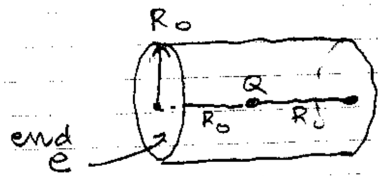
$\rightarrow E(r)_{r>r_2} = \dfrac{r_1^2\sigma_1 + r_2^2\sigma_2}{r^2\epsilon_0}$

d) $E_{r>r_2} = 0$ only when $r_1^2\sigma_1 + r_2^2\sigma_2 = 0$

or $\sigma_1 = -\dfrac{r_2^2}{r_1^2}\sigma_2$

e) $E_{r_1<r<r_2} = 0$ only when $\sigma_1 = 0$

22-35 (A tough one)



Gauss Law

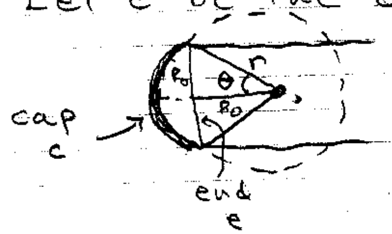
$$\frac{Q}{\epsilon_0} = \Phi_{tot} = \Phi_{side} + \Phi_{ends}$$

$$\Phi_{side} = \frac{Q}{\epsilon_0} - \Phi_{ends} = \frac{Q}{\epsilon_0} - 2\Phi_e$$

left end above

What is Φ_e?

Fit a sphere centered on Q over the end e. Let c be the "cap" that fits over e.



radius of sphere $= \sqrt{2} R_0$

angle $\theta = 45°$

First: No charge between end e and cap c. So Gauss's law says

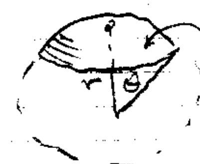
$$\Phi_e = \Phi_c$$

But because c is on a sphere with center at charge Q

$$\Phi_c = \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2} A_c$$

area of the cap

The area of a cap is simple (but I realize that you might not know it!)



area $= 2\pi r^2 (1-\cos\theta)$

So for us, with $r = \sqrt{2}R_0$ and $\theta = 45°$

$$\Phi_c = \frac{Q}{4\pi\epsilon_0} \frac{1}{(\sqrt{2}R_0)^2} \left(2\pi (\sqrt{2}R_0)^2\right)(1-\cos 45°)$$

$$= \frac{Q}{2\epsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$$

and

$$\Phi_{side} = \frac{Q}{\epsilon_0} - 2\Phi_e = \frac{Q}{\epsilon_0} - 2\Phi_c$$

$$= \frac{Q}{\epsilon_0\sqrt{2}}$$

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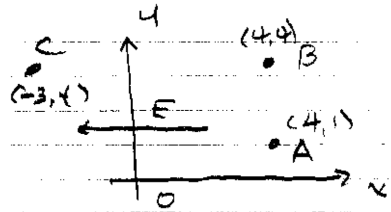

$\vec{E} = -300 \frac{N}{C} \vec{i}$
(constant)

$$V_b - V_a = -\int_a^b \vec{E} \cdot \vec{dl}$$

a) $V_{BA} = V_B - V_a = -\int_A^B \vec{E} \cdot \vec{dl}$

here $\vec{dl} = dy\,\vec{j}$

then $\vec{E} \cdot \vec{dl} = -E\,dy\,\underbrace{\vec{i} \cdot \vec{j}}_{\cos 90^\circ = 0}$

$V_{BA} = 0$ $\qquad = 0$

b) $V_{CB} = V_C - V_B = -\int_B^C \vec{E} \cdot \vec{dl}$

$= (-E) \int_4^{-3} dx\,(-\vec{i}) \cdot \vec{i}$

$= -E \int_{-3}^{4} dx$

$= -(300 \frac{N}{C})\,7\,m$

$= -2100 \frac{Nm}{C}$

c) $V_{CA} = V_C - V_A = -\int_A^C \vec{E} \cdot \vec{dl}$

The best way is to use

$V_{CA} = V_C - V_A$

$= V_C - V_B - (V_B - V_A)$

$= V_{CB} - V_{BA}$

$= -2100 \frac{Nm}{C} - \cancel{0}$

This works because the electric force is conservative (in static case).

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constant charge density is

$$\rho = \frac{Q}{\left(\frac{4}{3}\pi r_0^3\right)} = \frac{3Q}{4\pi r_0^3}$$

charge inside Gaussian sphere of radius r (with same center as charged sphere) is:

$$Q(r) = \rho \cdot \left(\tfrac{4}{3}\pi r^3\right) = Q\frac{r^3}{r_0^3}$$

from Gauss:

$$E(r) = \frac{1}{4\pi r^2}\left[Q\frac{r^3}{r_0^3}\right]\frac{1}{\epsilon_0} = \frac{Q}{4\pi\epsilon_0}\frac{r}{r_0^3} \quad \text{for } r < r_0$$

and

$$E(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \quad \text{for } r > r_0$$

Get $V(r)$ from

$$V_\infty - V_r = -\int_r^\infty E\,dr$$

See Ex 23-3 p. 597

and take $V_\infty = 0$

so:

$$V_r = \int_r^\infty E\,dr$$

a) for $r > r_0$

$$V_r = \int_r^\infty \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,dr = \frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\Bigg|_r^\infty = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \quad \text{(just like point charge)}$$

b) for $r < r_0$

$$V_r = \int_r^\infty E(r)\,dr = \int_{r_0}^\infty \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,dr + \int_r^{r_0}\frac{Q}{4\pi\epsilon_0}\frac{r}{r_0^3}\,dr$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0} + \frac{Q}{4\pi\epsilon_0}\frac{r_0^2 - r^2}{2r_0^3}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0}\left(1 + \frac{1}{2} - \frac{r^2}{2r_0^2}\right)$$

$$= \frac{Q}{8\pi\epsilon_0 r_0}\left(3 - r^2\right)$$

Plots are in solns. in book