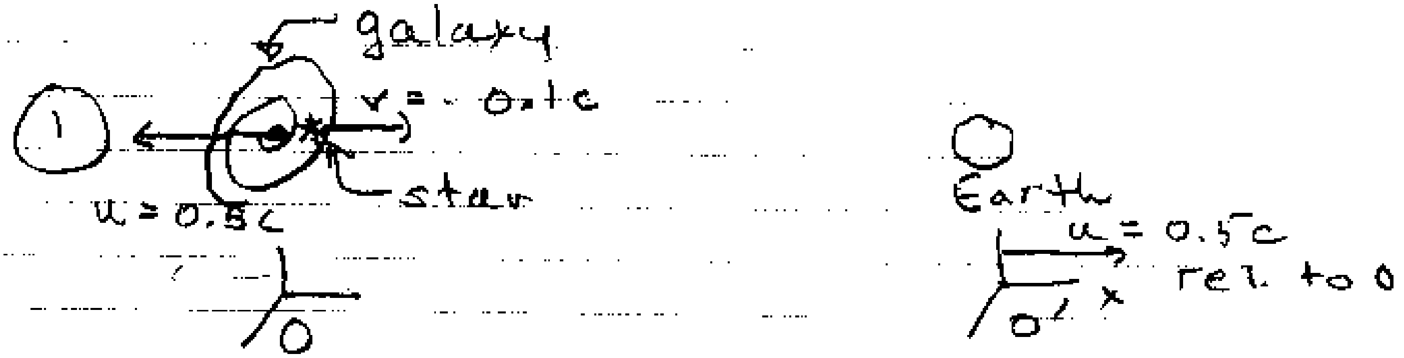
SOLUTIONS, MID-TERM I



vel. of star v' in rest frame of earth $= \dfrac{v-u}{1-uv/c^2}$ Krane: Eq. (2.28a)

$$= \frac{0.1c - 0.5c}{1-(0.5c)(0.1c)/c^2}$$

$$= \frac{-0.4c}{0.95} = -0.42c$$

in $-x$ direction

ie away from earth

b) light "red-shifted", according to longitudinal Doppler shift. Star moves away $\rightarrow \nu' < \nu$, $\lambda' > \lambda$

Krane Eq (2.20) $\quad \nu' = \nu\sqrt{\dfrac{1-v_s/c}{1+v_s/c}}$ $\quad v_s$ = vel of source away from observer

ν' ← frequency

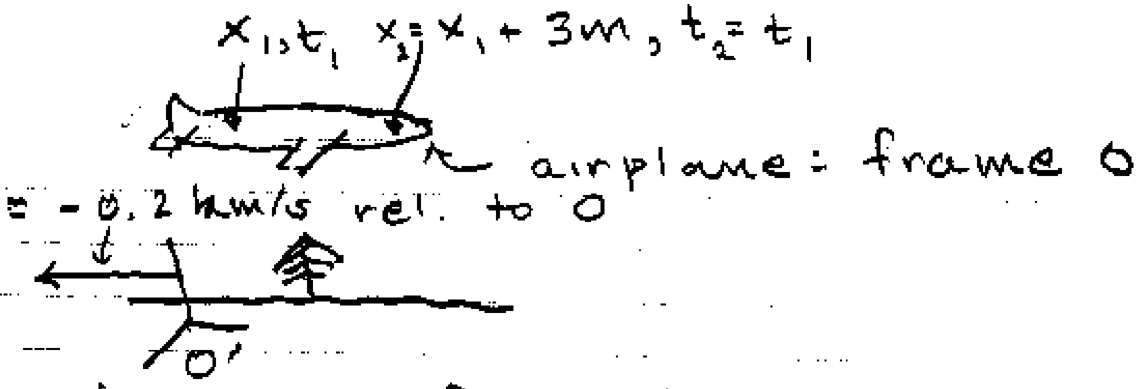
$$= \frac{c}{\lambda}\sqrt{\frac{1-(+0.42)}{1+(+0.42)}}$$

$$= \frac{3\times10^{8}\,\frac{m}{s}}{5\times10^{-7}\,m}\sqrt{\frac{0.58}{1.42}}$$

$$= \cancel{0.383\times10^{15}}\ s^{-1}$$

$$= 3.83\times10^{14}\ Hz$$

2)



Lorentz transformation:

$$t_2' = \frac{t_2 - (u/c^2)\, x_2}{\sqrt{1 - u^2/c^2}}$$

$$t_1' = \frac{t_1 - (u/c^2)\, x_1}{\sqrt{1 - u^2/c^2}}$$

$u = -0.2$ km/s
$t_1 = t_2$, $x_2 - x_1 = 30$ m

$$\Rightarrow t_2' - t_1' = -\frac{(-0.2\,\text{km/s}\,/c^2)\, 30\,\text{m}}{\sqrt{1 - \left(\frac{0.2\,\text{km/s}}{c}\right)^2}}$$

$$= \frac{+(2/9)\times 10^{-14}(\text{s/m}) \cdot 30\,\text{m}}{\sqrt{1 - \frac{4}{9}\times 10^{-18}}}$$

utterly negligible

$$\approx \frac{60}{9} \times 10^{-14}\,\text{s}$$

$$\approx 6.67 \times 10^{-14}\,\text{s}$$

the event at the back occurs first (but not by much!)

3) initial

$$P \longrightarrow \quad \longleftarrow P$$

$$v = 0.99\,c$$

final $\nearrow \Delta$ $\quad K_\Delta$?

$\Delta \swarrow$

Energy conservation

$$E_i = E_f$$

$E_i =$

$$2\times 6.63\times10^{3}\,\text{MeV} = \frac{2m_pc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = 2K_\Delta + 2m_\Delta c^2$$

$$K_\Delta = \frac{m_pc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - m_\Delta c^2$$

in Joules

$$6.63\times10^{9}\,\text{eV}\times1.6\times10^{-19}\,\frac{\text{J}}{\text{eV}}$$

$$= 1.06\times10^{-9}\,\text{J}$$

$$1-(1-10^{-2})^2 \simeq 2\times10^{-2}$$

$$= \frac{938\ \text{MeV}}{\sqrt{2\times10^{-2}}} - 1240\ \text{MeV}$$

$$= \left(\frac{9.38\times10^{3}}{\sqrt{2}} - 1240\right)\text{MeV}$$

$$= (6633 - 1240)\,\text{MeV}$$

$$= 5393\ \text{MeV}$$

4) This is the same configuration as the Davison - Germer experiment and maxima are given by

$$n\lambda = d \sin\phi$$

$$\sin\phi = \left(\frac{\lambda}{d}\right) n \leftarrow \text{must be } \leq 1$$

$$= \left(\frac{1\ \cancel{nm}}{2\ n\cancel{m}}\right) n$$

$$= 0.5\, n$$

maxima at

$\sin\phi = 0$, $n = 0$ (reflection)

$\sin\phi = 0.50$, $n = 1$ $\quad \phi = 30°$

$\sin\phi = 1$, $n = 2$ $\quad \phi = 90°$ (at the surface)

5) Compton scattering $\lambda = 10^{-2}$ nm

$$\lambda' - \lambda = \lambda_c (1 - \cos\theta)$$

$$\lambda_c = \frac{h}{m_e c} \simeq \frac{4.1 \times 10^{-15} \text{ eV}\cdot\text{s} \times 3 \times 10^{17} \frac{\text{nm}}{\text{s}}}{0.511 \times 10^{6} \text{ eV}}$$

$$\simeq \frac{3 \times 4.1}{0.5} \times 10^{-4} \text{ nm}$$

a) $\theta = 180°$: $\lambda_c \simeq 2.4 \times 10^{-3}$ nm

$$\lambda' - \lambda = 2\lambda_c$$

$$\lambda' = \lambda + 2\lambda_c = 1.48 \times 10^{-2} \text{ nm}$$

$$E' = h\nu' = \frac{hc}{\lambda'} = \frac{hc}{\lambda + 2\lambda_c}$$

$$p' = \frac{E}{c} = \frac{1}{c} \frac{hc}{\lambda + 2\lambda_c}$$

answer in eV

$$= \frac{1}{c} \frac{1.24 \times 10^{3} \text{ eV } \cancel{\text{nm}}}{1.48 \times 10^{-2} \cancel{\text{nm}}}$$

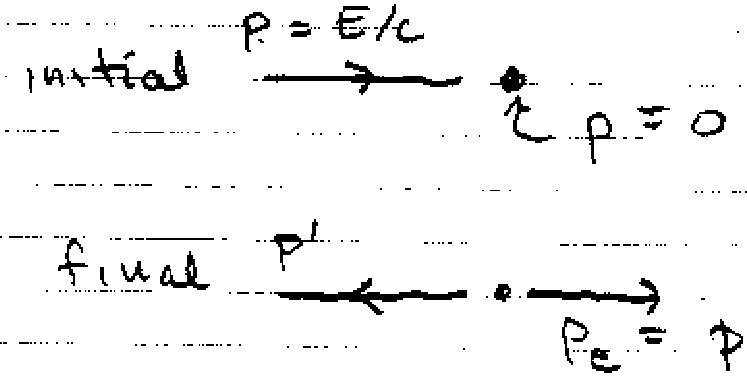
$$\simeq \frac{1.24}{1.48} \times 10^{5} \text{ eV}/c$$

$$\simeq 0.84 \times 10^{5} \frac{\text{eV}}{c} \times \left(1.6 \times 10^{-19} \frac{\text{J}}{\text{eV}}\right)$$

$$\simeq 1.3 \times 10^{-14} \text{ J}/c \simeq 0.4 \times 10^{-22} \text{ kg} \frac{\text{m}}{\text{s}}$$

redo here

b) For back scattering



$$p = \frac{E}{c} = \frac{1}{c}\left(\frac{hc}{\lambda}\right)$$

$$= \frac{1}{c}\,\frac{1.24\times10^{3}\,eV\,nm}{10^{-3}\,nm}$$

$$= \frac{1}{c}\,1.24\times10^{3}\,eV$$

$$=$$

$$p_e = p + p'$$

$$= \frac{E}{c} + p' = \frac{hc}{\lambda c} + p'$$

$$= \frac{1}{c}\left(1.24\times10^{5}\,eV + 0.84\times10^{5}\,eV\right)$$

$$= \frac{1}{c}\left(2.08\times10^{5}\,eV\right)$$

$$\lambda_e = \frac{h}{p} = \frac{hc}{2.08\times10^{5}\,eV} = \frac{1.24\times10^{3}\,eV\,nm}{2.08\times10^{5}\,eV}$$

$$\approx 0.5\times10^{-2}\,nm$$

6) Bohr H atom

$$E_n = -13.6\ \text{eV}\ \frac{1}{n^2}$$

$$E_\gamma = E_n - E_1$$

$$= -13.6\ \text{eV}\left(\frac{1}{n^2} - 1\right)$$

$$= 13.6\ \text{eV}\left(\frac{n^2-1}{n^2}\right)$$

$$h\nu_\gamma = 13.6\ \text{eV}\left(\frac{n^2-1}{n^2}\right)$$

$$\nu_\gamma = \frac{13.6\ \text{eV}}{4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}}\left(\frac{n^2-1}{n^2}\right)$$

$$= 3.29 \times 10^{15}\ \text{s}^{-1}\left(\frac{n^2-1}{n^2}\right)$$

3 lowest

$2 \to 1$

$$\nu_{\text{lowest}} = 3.29 \times 10^{15}\ \text{s}^{-1}\left(\frac{3}{4}\right)$$

$$= 2.46 \times 10^{15}\ \text{s}^{-1}$$

$3 \to 1$

$$\nu = 3.29 \times 10^{15}\ \text{s}^{-1}\left(\frac{7}{8}\right)$$

$$= 2.88 \times 10^{15}\ \text{s}^{-1}$$

$4 \to 1$

$$\nu = 3.08 \times 10^{15}\ \text{s}^{-1}$$

Can also do this by

$$\lambda = \lambda_{\text{limit}}^{(\text{Lyman})} \left(\frac{n^2}{n^2 - 1} \right)$$

$$= \frac{1}{R_\infty} \left(\frac{n^2}{n^2 - 1} \right)$$

and then find

$$\nu = c/\lambda$$

$$v'(1 - vu/c^2) = v - u$$

$$v' + u = v(1 + v'u/c^2)$$

$$\frac{v' + u}{1 + v'u/c^2} = v$$

if earth frame is O, then
$u = -0.5c$, $v' = +0.1c$

$$v = \frac{(0.1 - 0.5)c}{1 + (0.1)(-0.5)}$$

$$= \frac{0.4}{0.95}$$