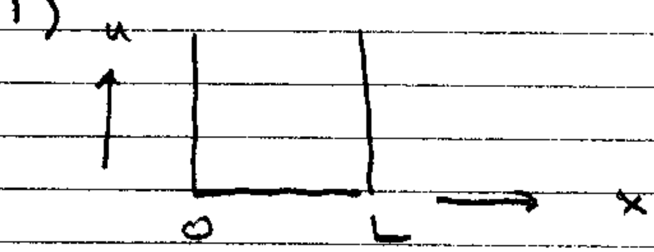
PHYS 251 SPR '02

MIDTERM II : SOLUTIONS

1)



For particle in a box

$$E_n = \frac{\pi^2 \hbar^2}{2mL^2} n^2 = \frac{(1/4)(hc)^2}{2\, mc^2 L^2} n^2$$

so

$$L = \frac{(hc)^2}{8\, mc^2} \frac{1}{E_n} n^2$$

for us, $n = 5$, $E_5 = 10$ eV

$$mc^2 = 938 \text{ MeV} = 9.38 \times 10^8 \text{ eV}$$

$$L = \left(\frac{(1.24 \times 10^3\ \cancel{\text{eV}} \cdot \text{nm})^2}{8 \cdot 9.38 \times 10^8\ \cancel{\text{eV}}} \cdot \frac{25}{10\ \cancel{\text{eV}}} \right)^{1/2}$$

$$= 0.08 \times 10^{(6-8)/2} \text{ nm}$$

$$= .8 \times 10^{-3} \text{ nm}$$

(2) Turning point

$$\cancel{\tfrac{1}{2}}\hbar\omega_0 = \cancel{\tfrac{1}{2}}kx_0^2$$

$$x_0 = \sqrt{\hbar\omega_0/k}$$

Maximum momentum

$$\cancel{\tfrac{1}{2}}\hbar\omega_0 = \frac{p_0^2}{\cancel{2}m}$$

$$\sqrt{\cancel{2}m\hbar\omega_0} = p_0$$

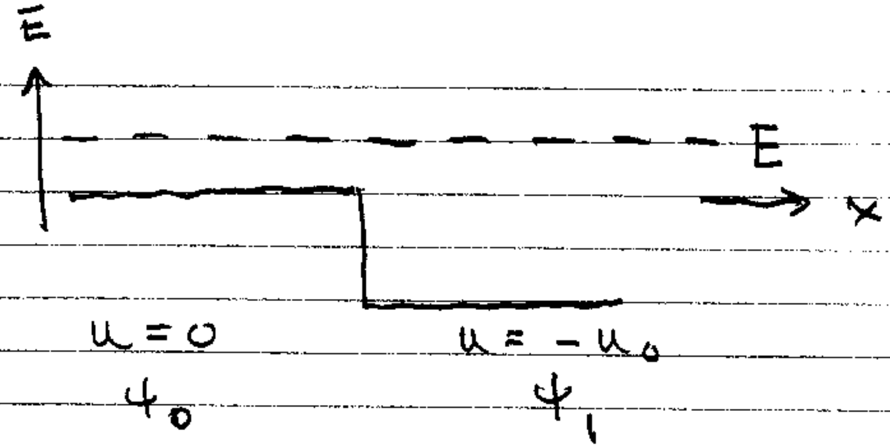
Product

$$p_0 x_0 = \sqrt{\hbar\underbrace{\omega_0/k}_{1/\sqrt{km}}}\ \sqrt{m\hbar\underbrace{\omega_0}_{\sqrt{\frac{k}{m}}}}$$

$$= \sqrt{\hbar^2\frac{1}{\sqrt{km}}m\sqrt{\frac{k}{m}}} = \hbar$$

The particle is constrained to be in a region of size x_0, so it must have a momentum that is of order $\hbar/x_0$ by the uncertainty principle.

3)



a) $x<0$ $\quad \psi_0 = Ae^{ikx} + Be^{-ikx}$

incident $\quad$ refected

$x>0$ $\quad \psi_1 = Ce^{ik'x}$

transmitted

$k = \frac{1}{\hbar}\sqrt{2mE}$ $\qquad k' = \frac{1}{\hbar}\sqrt{2m(E+u_0)}$

this is $E-u$

b) Continuity:

$\psi_0(0) = \psi_1(0)$

$A + B = C$

$\frac{B}{A} + 1 = \frac{C}{A}$ (1)

$\psi_0'(0) = \psi_1'(0)$

$ik(A-B) = ik'C$

$1 - \frac{B}{A} = \frac{k'}{k}\frac{C}{A}$ (2)

$\rightarrow \frac{B}{A}$ and $\frac{C}{A}$

Substitute (1) in (2)

$$1 - \left(\frac{C}{A} - 1\right) = \frac{k'}{k}\frac{C}{A}$$

$$2 = \left(\frac{k'}{k} + 1\right)\frac{C}{A}$$

$$\Rightarrow \left(\frac{C}{A}\right)^2 = \left(\frac{2k}{k'+k}\right)^2$$ is intensity of transmitted wave

Then, back in (1)

$$\frac{B}{A} = \frac{C}{A} - 1 = \frac{2k}{k'+k} - 1 = \frac{k-k'}{k+k'}$$

$$\left(\frac{B}{A}\right)^2 = \left(\frac{k-k'}{k+k'}\right)^2$$ is intensity of reflected wave

4) $(u(r))_{avg}$

$$= -\frac{1}{4\pi\varepsilon_0}\int_0^{\infty} P(r)_{n=1,\ell=0} \frac{1}{r}\, dr$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{4}{a_0^3}\int_0^{\infty} r^2 e^{-2r/a_0} \cdot \frac{1}{r}\, dr$$

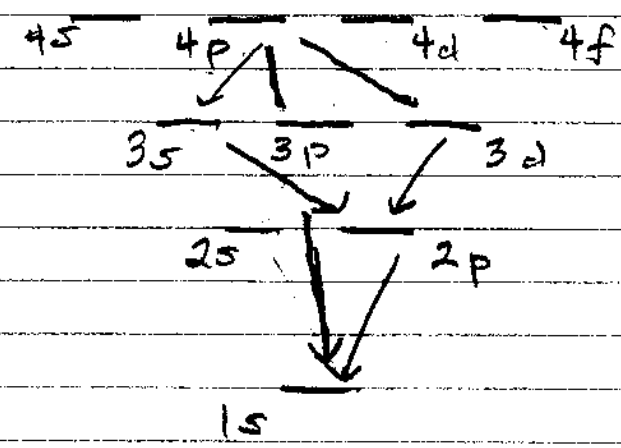
$$= -\frac{1}{4\pi\varepsilon_0}\frac{4}{a_0^3}\int_0^{\infty} e^{-2r/a_0}\, r\, dr$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{4}{a_0^3}\frac{1!}{(2/a_0)^2}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{1}{a_0}$$

Selection rule is $\Delta l = \pm 1$

5)



$$4p \xrightarrow{+1} 3d \xrightarrow{-1} 2p \xrightarrow{-1} 1s \quad (\Delta l =)$$

$$4p \xrightarrow{-1} 3s \xrightarrow{+1} 2p \xrightarrow{-1} 1s$$

$$4p \xrightarrow{-1} 1s$$

6) In a closed subshell, all values of m_l and m_s are used up for fixed n and l. So for every $m_s = 1/2$, we have $m_s = -1/2$, by the Pauli principle.