

PHY 251 SPR 02
HW # 11

Compute $\langle U \rangle$ in H 1s state

7.18

$$\langle U \rangle = \int_0^\infty U(r) P(r) dr$$

$$= -\frac{e^2}{4\pi\epsilon_0} \int_0^\infty \frac{1}{r} \cdot r^2 \left(\frac{2}{a_0^{3/2}} e^{-r/a_0} \right)^2 dr$$

$$= -\frac{e^2}{4\pi\epsilon_0} \frac{4}{a_0^3} \int_0^\infty r e^{-2r/a_0} dr$$

$$= -\frac{e^2}{\pi\epsilon_0 a_0^3} \frac{1!}{(2/a_0)^2}$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{1}{a_0}$$

↑ potential for
electron in orbit
of radius a_0

9.12 Use the figure and

$$k = \frac{2(E - E_{\min})}{(R - R_{eq})^2} \quad \text{define} \quad \frac{2\Delta E}{\Delta R^2}$$

estimate

$$\Delta E = 0.25 \text{ eV}$$

for

$$\Delta R = 0.05 \text{ nm}$$

or

$$k \approx \frac{0.50 \text{ eV}}{(0.05)^2 \text{ nm}^2} = \frac{5 \times 10^{-1}}{2.5 \times 10^{-3}} \text{ eV nm}^{-2}$$

$$k \approx 2 \times 10^2 \frac{\text{eV}}{\text{nm}^2} \approx 2 \times 10^{20} \frac{\text{eV}}{\text{m}^2}$$

which gives

$$\nu = \frac{1}{2\pi} \sqrt{\frac{k c^2}{m c^2}}$$

$$m = \frac{m_{\text{Na}} m_{\text{Cl}}}{m_{\text{Na}} + m_{\text{Cl}}} = \frac{22 \times 35}{22 + 35} \text{ u} \approx 14 m_p$$

$$\nu \approx \frac{1}{2\pi} \sqrt{\frac{2 \times 10^{20} \frac{\text{eV}}{\text{m}^2} \times 9 \times 10^{16} \frac{\text{m}^2}{\text{s}^2}}{14 \times 10^9 \text{ eV}}}$$

$$\approx \frac{1}{2\pi} \sqrt{\frac{18}{14} \times 10 \times 10^{26}} \text{ Hz}$$

$$\approx \frac{1}{2\pi} 3.5 \times 10^{13} \text{ Hz}$$

$$\approx 5 \times 10^{12} \text{ Hz}$$

$$\lambda = \frac{c}{\nu} \approx 6 \times 10^{-5} \text{ m}$$

infrared (see figure on p. 6)

$$h\nu \approx (4 \times 10^{-15} \text{ eV} \cdot \text{s}) (5 \times 10^{12} \text{ Hz})$$

$$\approx 2 \times 10^{-2} \text{ eV}$$

$$h\nu \approx .004 \text{ eV}$$

at about 0.2 eV,
parabolic approx. fails
corresponds to about $n \approx 10$

9.13

The only difference between H_2 and HD is their reduced mass

$$m^{H_2} = \frac{m_p^2}{2m_p} = \frac{m_p}{2}$$

$$m^{HD} = \frac{m_p(2m_p)}{m_p + 2m_p} = \frac{2}{3}m_p = \frac{4}{3}m^{H_2}$$

R_{eq} is independent of m , since it is determined by electronic states.

The relevant formulas are

$$\nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

(k is also m -independent)

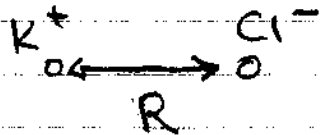
$$\rightarrow \nu_{HD} = \sqrt{\frac{3}{4}} \nu_{H_2}$$

Similarly

$$\frac{\hbar^2}{2m^{HD}R_{eq}^2} = \frac{3}{4} \frac{\hbar^2}{2m^{H_2}R_{eq}^2}$$

9.3 $E_{ion}^K = 4.34 \text{ eV}$

$E_{aff}^{Cl} = 3.06 \text{ eV}$



find R such that

$$U(R) = E_{aff}^K - E_{ion}^K$$

$$= (3.06 - 4.34) \text{ eV}$$

$$= -1.28 \text{ eV}$$

$$U(R) = -\frac{e^2}{4\pi\epsilon_0} \frac{1}{R} = -1.28 \text{ eV}$$

$$R = (1.28 \text{ eV})^{-1} \left(\frac{e^2}{4\pi\epsilon_0} \right)$$

$$\approx 1.440 \text{ eV nm}$$

$$= 1.13 \text{ nm}$$