PHY 251 SPR 02 HW 12 (THE LAST ONE)

(10.9)

- Noninteracting H atoms in $2p$ state
- Magnetic field: $B = 5.0$ T
- Room temperature ($kT = 1/40$ eV)
- Neglect spin.

(a) Fractions of atoms in $m_\ell = +1, 0, -1$
From Krane (7.22), the energy of the atoms is

$$E(m_\ell) = E_\mathrm{H}(2p) + m_\ell \mu_\mathrm{B} B$$

where $E_\mathrm{H}(2p)$ is the energy of an H atom in the $2p$ state (which we won't need).

The fraction of atoms with m_ℓ at temperature T is determined by the ratio of Boltzmann factors.

$$f(m_\ell) = \frac{\mathrm{e}^{-E(m_\ell)/kT}}{\mathrm{e}^{-E(1)/kT} + \mathrm{e}^{-E(0)/kT} + \mathrm{e}^{-E(-1)/kT}}$$

Now divide top and bottom by $\exp[E_\mathrm{H}(2p)/kT]$ to get:

$$f(m_\ell) = \frac{\mathrm{e}^{-m_\ell \mu_\mathrm{B} B/kT}}{\mathrm{e}^{-\mu_\mathrm{B} B/kT} + 1 + \mathrm{e}^{-\mu_\mathrm{B} B/kT}}$$

The rest of (a) is substitution.

(b) Ratios of intensities:
Here use the selection rule

$$\Delta m_\ell = 0, \pm 1$$

given in Krane (7.25). All three values of m_ℓ in the 2p state can make the transition to the $1s$ state (with $m_\ell = 0$), so we expect the ratios of the intensities to be the same as the ratios of atoms in each level.

(10.13)

(a) Number of photons of energy E, per unit energy and volume:

We get this from Krane (10.27) – (10.29).

$$n(E) = \frac{p(E)}{L^3} = \pi \left(\frac{1}{\hbar c \pi}\right)^3 E^2 \frac{1}{e^{E/kT} - 1}$$

(b) Total number of photons:

$$\begin{aligned} N_{\rm tot} &= \int_0^\infty n(E) dE \\ &= \pi \left(\frac{1}{\hbar c \pi}\right)^3 \int_0^\infty E^2 \frac{1}{e^{E/kT} - 1} \, dE \end{aligned}$$

Now let $x = E/kT$, so that $E = (kT)x$ and $dE = (kT)dx$, and use $\pi\hbar = h/2$, to get the quoted result:

$$N_{\rm tot} = 8\pi \left(\frac{kT}{hc}\right)^3 \int_0^\infty \frac{x^2}{e^x - 1} \, dx$$

(c) The number of photons per cubic centimeter, using that the integral is about 2.404. Take T as 3 K and 300 K. Putting in the constants gives:

$$N_{\rm tot} = 8\pi \left(\frac{8.617 \times 10^{-5} eVK}{1.240 \times 10^3 eV \times 10^{-7} cm}\right)^3 T^3 \ (2.404)$$

Then substitute for kT.

(10.21)

A metal with Fermi energy $E_F = 3.00$ eV, the probability to find an electron with energy between 5.00 and 5.10 eV, for (a) T=295 K and (b) T=2500 K.

Consulting section 10.7 of Krane, the number of electrons is given by integrating the number density (number per unit energy) between 5 and 5.1 eV, which we'll call $N(\Delta E)$, and then dividing by the total number, N. For the number density, we use $p(E)$, equation (10.36), and for the total number we use (10.38):

$$\begin{aligned}
\frac{n(\Delta E)}{N} &= \frac{\int_{5.00eV}^{5.10eV} p(E)dE}{N} \\
&= \frac{3}{2E_F^{3/2}} \int_{5.00eV}^{5.10eV} \frac{E^{1/2}}{e^{(E-E_F)/kT}+1} dE \\
&\sim \frac{3}{2E_F^{3/2}} (5.00eV)^{1/2} \int_{5.00eV}^{5.10eV} e^{-(E-E_F)/kT} dE \qquad (1)
\end{aligned}$$

In the last step, we've used two facts: (i) that $E^{1/2}$ is almost constant for E between 5 and 5.1 eV, and (ii) that $e^{(E-E_F)/kT}$ is much, much larger than 1 in this range. These approximations are quite good. Now we have to do the E integral, which isn't too hard,

$$\begin{aligned}
\frac{n(\Delta E)}{N} &= \frac{3}{2E_F^{3/2}} (5.00eV)^{1/2} e^{E_F/kT} \int_{5.00eV}^{5.10eV} e^{-E/kT} dE \\
&= \frac{3kT}{2E_F^{3/2}} (5.00eV)^{1/2} (-e^{-(E-E_F)/kT}) |_{5.00eV}^{5.10eV}
\end{aligned}$$

Finally, substitute for kT and E_F.