PHY 251 SPR 02

Hw #6

4.28, 5.2, 5.3, 5.7 Rec 5.9, 5.12

4-28 (The Fig is 4.22 in my book, not 4.21)

$\lambda_1 = 1$, $\lambda_2 = 10/9$

$\rightarrow k_1 = 2\pi$, $k_2 = \frac{18\pi}{10} = \frac{9}{5}\pi$

$v_1 = 3 \rightarrow \frac{\omega_1}{k_1} = 3 \rightarrow \omega_1 = 6\pi$

$v_2 = 2.5 \rightarrow \frac{\omega_2}{k_2} = 2.5 \rightarrow \omega_2 = 2.5 \cdot \frac{9}{5}\pi = \frac{9}{2}\pi$

then

$\Delta k = (2 - \frac{9}{5})\pi$

$\Delta\omega = (6 - \frac{9}{2})\pi$

$v_g = \frac{\Delta\omega}{\Delta k} = \frac{6 - \frac{9}{2}}{2 - \frac{9}{5}} = \frac{3/2}{1/5} = \frac{15}{2} = 7.5$ ✓

5.2 $y_-(x) = -A\cos\left(\frac{2\pi x}{\lambda} + \frac{\pi}{6}\right) \quad x<0$

$y_+(x) = -B\cos\left(\frac{2\pi x}{\lambda/2} + \phi'\right) \quad x>0$

condition at $x=0$

$y_-(0) = y_+(0)$

$A\cos(\pi/6) = B\cos(\phi')$ ($\pi/6 = 30^\circ$)

$y'_-(0) = y'_+(0)$

$A\frac{2\pi}{\lambda}\sin(\pi/6) = B\frac{4\pi}{\lambda}\sin(\phi')$

$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \sin 30^\circ = \frac{1}{2}$

$A\frac{\sqrt{3}}{2} = B\cos\phi' \qquad (1)$

$\frac{A}{2\lambda} = B\frac{2}{\lambda}\frac{\sqrt{3}}{2}\sin\phi \qquad (2)$

(1)/(2) $\rightarrow$

$\sqrt{3}\lambda = \frac{\lambda}{\sqrt{3}}\cot\phi$

$3 = \cot\phi$

(1) $\rightarrow$ $A\frac{\sqrt{3}}{2} = B\cos(\cot^{-1}3)$

$B = \frac{A\sqrt{3}}{2}\frac{1}{\cos(\cot^{-1}3)}$

5.3 $E_{n=1} = 4.4\,eV$

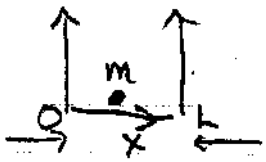
$$E_1 = \frac{\hbar^2 \pi^2}{2mL^2}$$ Eq (5.25) p 148

If $L \to 2L$

E_1 (all E_n, in fact) decrease by 4

$$E_1 \to \frac{4.4}{4}\,eV = 1.1\,eV$$

5.7



Ground state

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

Probabilities

$$P_a = \int_0^{L/3} dx\, \psi^2(x)$$

$$= \frac{2}{L}\int_0^{L/3} \sin^2 \frac{\pi x}{L}\, dx$$

$y = \pi x/L$

$$= \frac{2}{4}\frac{L}{\pi}\int_0^{\pi/3} \sin^2 y\, dy = \frac{2}{\pi}\int_0^{\pi/3} \frac{(1-\cos 2y)}{2}\, dy$$

$$= \frac{2}{\pi}\left[\frac{1}{2}y\Big|_0^{\pi/3} - \frac{1}{2}\cdot\frac{1}{2}\sin 2y\Big|_0^{\pi/3}\right]$$

$$P_a = \frac{2}{\pi}\left[\frac{\pi}{6} - \frac{1}{4}\sin\frac{2\pi}{3}\right]$$

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128

$$= \frac{2}{\pi}\left[\frac{\pi}{6} - \frac{1}{4}\frac{\sqrt{3}}{2}\right]$$

$$= 0.1955 \quad \text{(amazing!)}$$

$$P_b = \frac{2}{L}\int_{L/3}^{2L/3} dx \sin^2\frac{\pi x}{L}$$

$$= \frac{2}{\pi}\left[\frac{1}{2}y\Big|_{\pi/3}^{2\pi/3} - \frac{1}{4}\sin 2y\Big|_{\pi/3}^{2\pi/3}\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{2}\frac{\pi}{3} - \frac{1}{4}\left(\sin\frac{4\pi}{3} - \sin\frac{2\pi}{3}\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{\pi}{6} - \frac{1}{4}\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)\right]$$

similarly $= \frac{2}{\pi}\left[\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right] = 0.6089$

0

$$P_c = \frac{2}{\pi}\left[\frac{1}{2}\left(\pi - \frac{2\pi}{3}\right) - \frac{1}{4}\left(\sin 2\pi - \sin\frac{4\pi}{3}\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{\pi}{6} - \frac{1}{4}\left(-\left(-\frac{\sqrt{3}}{2}\right)\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right] = 0.1955$$

notice $P_a + P_b + P_c = 1.0000$