PHY 251 SPR 02

Hw Set # 7

Sch. Eq:

5.25 $$-\frac{\hbar^2}{2m}\frac{d^2\psi_1}{dx^2}+\frac{1}{2}kx^2\psi_1=E_1\psi$$

$$\psi_1 = Axe^{-ax^2}$$

cancel all A's

$$\rightarrow -\frac{\hbar^2}{2m}\frac{d}{dx}\left(\frac{d}{dx}xe^{-ax^2}\right)+\frac{1}{2}kx^2\left(xe^{-ax^2}\right) = Exe^{-ax^2}$$

$$\rightarrow -\frac{\hbar^2}{2m}\frac{d}{dx}\left(e^{-ax^2}+x(-2ax)e^{-ax^2}\right) + \frac{1}{2}kx^3e^{-ax^2} = Exe^{-ax^2}$$

$$\rightarrow -\frac{\hbar^2}{2m}\frac{d}{dx}\left[(1-2ax^2)e^{-ax^2}\right] + \frac{1}{2}kx^3e^{-ax^2} = Exe^{-ax^2}$$

$$\rightarrow -\frac{\hbar^2}{2m}\left[(-2ax-4ax+x(-2ax)^2)\cancel{e^{-ax^2}}\right] + \frac{1}{2}kx^3\cancel{e^{-ax^2}} = Ex\cancel{e^{-ax^2}}$$

$$\rightarrow -\frac{\hbar^2}{2m}\left[-6ax+4a^2x^3\right]+\frac{1}{2}kx^3 = Ex$$

x terms and x^3 terms must match independently

x terms:

$$\frac{3\hbar^2}{m} a = E$$

and

x^3 terms:

$$\frac{2\hbar^2}{m} a^2 = \frac{1}{2} k$$

$$\downarrow$$

$$a = \frac{1}{2\hbar}\sqrt{km} = \frac{1}{2\hbar} m \sqrt{\frac{k}{m}} = \frac{m\omega_0}{2\hbar}$$

same as for $n=0$ state (Krane (5.42))

then

$$E = \frac{3\hbar^2}{m} a$$

$$E = \frac{3}{2}\hbar\sqrt{\frac{k}{m}} = \frac{3}{2}\hbar\omega_0 \quad \text{since } \omega_0 = \sqrt{\frac{k}{m}}$$

this is $n=1$ in Eq. (5.45)

To find A:

$$1 = \int_{-\infty}^{\infty} |\psi(x)|^2 dx$$

$$= |A|^2 \int_{-\infty}^{\infty} x^2 e^{-2ax^2} dx$$

$$= |A|^2 (-1) \frac{d}{dy} \int_{-\infty}^{\infty} e^{-yx^2} dx \Big|_{y=2a}$$

$$= |A|^2 (-1) \frac{d}{dy} \sqrt{\frac{\pi}{y}} \Big|_{y=2a}$$

$$= \frac{|A|^2}{2} \frac{\sqrt{\pi}}{(2a)^{3/2}} = \frac{|A|^2\sqrt{\pi}}{2\left(\frac{1}{\hbar}\sqrt{km}\right)^{3/2}} \Rightarrow A^2 = \frac{2\left(\frac{1}{\hbar}\sqrt{km}\right)^{3/2}}{\sqrt{\pi}}$$

5.27 E_0 of oscillator $= 1.24\,eV$

$$E_0 = \tfrac{1}{2}\hbar\omega_0$$

second excited:

$$E_2 = (2 + \tfrac{1}{2})\hbar\omega_0 = \tfrac{5}{2}\hbar\omega_0$$

$\hookrightarrow$ $\Delta E_2 = E_2 - E_0$ excitation energy

$$= 2\hbar\omega_0$$

$$= 4E_0$$

second $= 4.96\,eV$

$$\Delta E_4 = E_4 - E_0$$

$$= 4\hbar\omega_0$$

$$= 8E_0$$

$$= 9.92\,eV$$

5-33 Start with 5.53

$$\psi_0(x) = A' e^{ik_0 x} + B' e^{-ik_0 x}$$

$$\psi_1(x) = C' e^{ik_1 x} + D' e^{-ik_1 x}$$

$$k_0 = \sqrt{\frac{2mE}{\hbar^2}} \qquad k_1 = \sqrt{\frac{2m}{\hbar^2}(E - u_0)}$$

Apply continuity for ψ, ψ' @ $x=0$
We did this in class
(notes: 15.6 – 15.7)

up to relations

$$\psi_0(0) = \psi_1(0)$$

$$A' + B' = C' \quad (1)$$

($D' = 0$ because incident wave is from $x < 0$)

$$A' k_0 - B' k_0 = C' k_1 \quad (2)$$

(1) in (2)

$$(A' - B') k_0 = (A' + B') k_1$$

$$A'(k_0 - k_1) = B'(k_0 + k_1)$$

$$\frac{k_0 - k_1}{k_0 + k_1} = \frac{B'}{A'} \xrightarrow[k_1 \to k_0]{} 0$$

and $\dfrac{C'}{A'} = 1 + \dfrac{B'}{A'} = \dfrac{2k_0}{k_0 + k_1} \xrightarrow[k_1 \to k_0]{} 1$

5-34 $E < U_0$ in

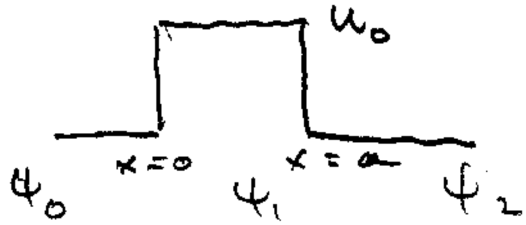

Fig 5.15

ψ illustrated in Fig 5.16

a) write down ψ_i's as expon.

b) identify 4 relations

c) incident particles from left → which coeff. is zero

follows class discussion