PHY682 Special Topics in Solid-State Physics: Quantum Information Science

Lecture time: 2:40-4:00PM Monday & Wednesday

Today:

1. Review of previous material
2. Finish Week 2 (From foundation to science-fiction teleportation)
3. Begin Week 3 material (Information is Physical)

Review: Quantum Teleportation

$$|\psi\rangle_1 \otimes |\Phi^+\rangle_{23} = \frac{1}{\sqrt{2}}(a|0\rangle + b|1\rangle) \otimes (|00\rangle + |11\rangle)_{23}$$

$$= \frac{1}{\sqrt{2}}(a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle)$$

$$|\Phi^{\pm}\rangle \equiv \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

$$|\Psi^{\pm}\rangle \equiv \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

measure in Bell basis

$$= \frac{1}{2}\{a(|\Phi^+\rangle + |\Phi^-\rangle) \otimes |0\rangle + a(|\Psi^+\rangle + |\Psi^-\rangle) \otimes |1\rangle + b(|\Psi^+\rangle - |\Psi^-\rangle) \otimes |0\rangle + b(|\Phi^+\rangle - |\Phi^-\rangle) \otimes |1\rangle\}$$

phase flip

$$= \frac{1}{2}\{\underbrace{|\Phi^+\rangle}_{12} \otimes (a|0\rangle + b|1\rangle) + \underbrace{|\Phi^-\rangle}_{12} \otimes (a|0\rangle - b|1\rangle) + \underbrace{|\Psi^+\rangle}_{12} \otimes (a|1\rangle + b|0\rangle) + \underbrace{|\Psi^-\rangle}_{12} \otimes (a|1\rangle - b|0\rangle)\}$$

$$I|\psi\rangle \qquad Z|\psi\rangle \qquad X|\psi\rangle \qquad iY|\psi\rangle$$

The unknown information a & b is preserved in the third particle, but depending on the outcome of the 'Bell-state' measurement in the basis of $\Phi^{\pm}$ & $\Psi^{\pm}$

Four possible outcomes, Alice informs Bob: (1) Φ^+ → apply identity (nothing); (2) Φ^- → apply Z to particle 3; (3) Ψ^+ → apply X to particle 3; (4) Ψ^- → apply –iY to particle 3

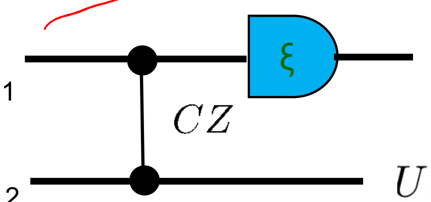
➔ Recover ψ at particle 3

Review: A variant---gate teleportation*

Controlled-Z gate and single-qubit measurement induces rotation

$$|\text{in}\rangle = (a|0\rangle + b|1\rangle)_1$$

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} = |+\rangle_2$$



+ or –
s=0 or 1

$U(\xi, s)|\text{in}\rangle_2$

$$CZ_{12} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

$$\boxed{U(\xi, s) = H\, e^{i\xi Z/2} Z^s}$$

single-qubit gate

later lecture

⇒ measurement-based QC.

The measurement basis ξ is defined via

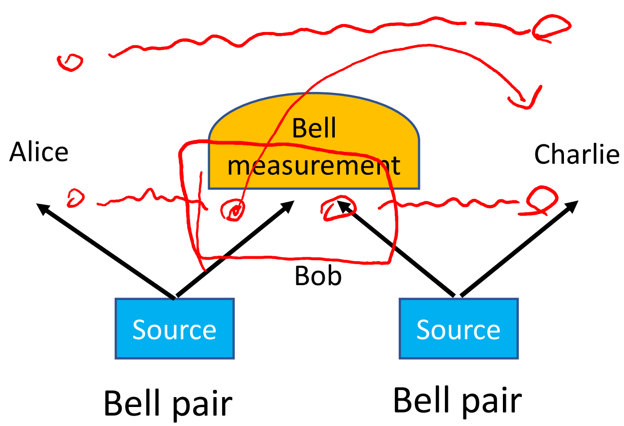
$$|\pm\xi\rangle = (e^{-i\xi/2}|0\rangle \pm e^{i\xi/2}|1\rangle)/\sqrt{2}$$

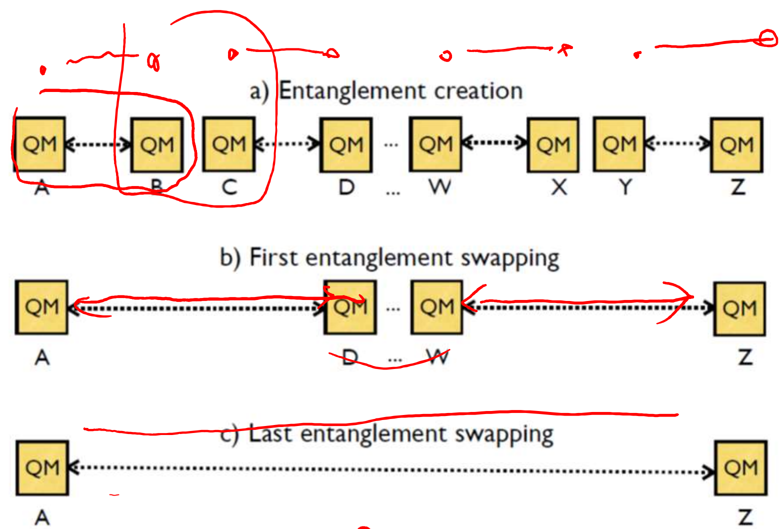
or the observable:

$$\boxed{\underbrace{\cos(\xi)\sigma_x}_{\equiv X} + \underbrace{\sin(\xi)\sigma_y}_{\equiv Y}} = \begin{pmatrix} 0 & e^{-i\xi} \\ e^{i\xi} & 0 \end{pmatrix}$$

Review: Entanglement swapping (via teleportation)

Imagine that Alice and Bob share an entangled pair and Bob and Charlie share another entangled pair. By performing the Bell-state measurement on Bob's two particles, Bob 'teleports' his entanglement with Charlie to Alice (or equivalently, Bob 'teleports' his entanglement with Alice to Charlie). This results in shared entanglement between Alice and Charlie.





form a quantum network

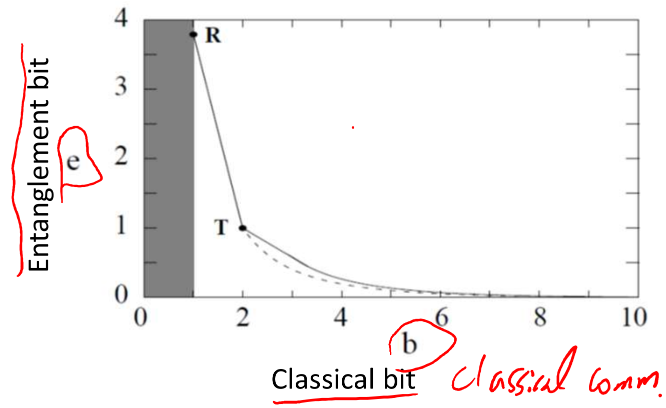
- Entanglement swapping is the basic protocol to establish entanglement between distant nodes (such as the Duan-Lukin-Cirac-Zoller scheme with atomic ensemble quantum memory)

Review: Remote state preparation*

It uses shared entanglement, e.g. the singlet state (which is antisymmetric):

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

[Bennet et al. PRL87, 077902 (2001)]

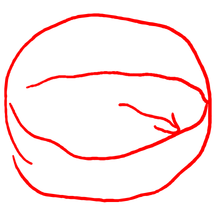


From the antisymmetry, one sees that for any single qubit state ψ and its orthogonal $\psi^\perp$:

$$|\Psi^-\rangle = e^{i\theta}\frac{1}{\sqrt{2}}(|\psi\rangle \otimes |\psi^\perp\rangle - |\psi^\perp\rangle \otimes |\psi\rangle) \longrightarrow$$

If Alice performs measurement on her particle in the basis $\{\psi,\psi^\perp\}$, with probability 1/2, she obtains $\psi^\perp$ and thus **prepares** Bob's state in ψ, and similarly with probability 1/2 prepares Bob's state in $\psi^\perp$

In the latter case, it is in general impossible for Bob to transform from in $\psi^\perp$ to ψ, except for 'equatorial states'



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle) \qquad |\psi^\perp\rangle = \frac{1}{\sqrt{2}}(|0\rangle - e^{i\phi}|1\rangle) = Z|\psi\rangle$$

Today:

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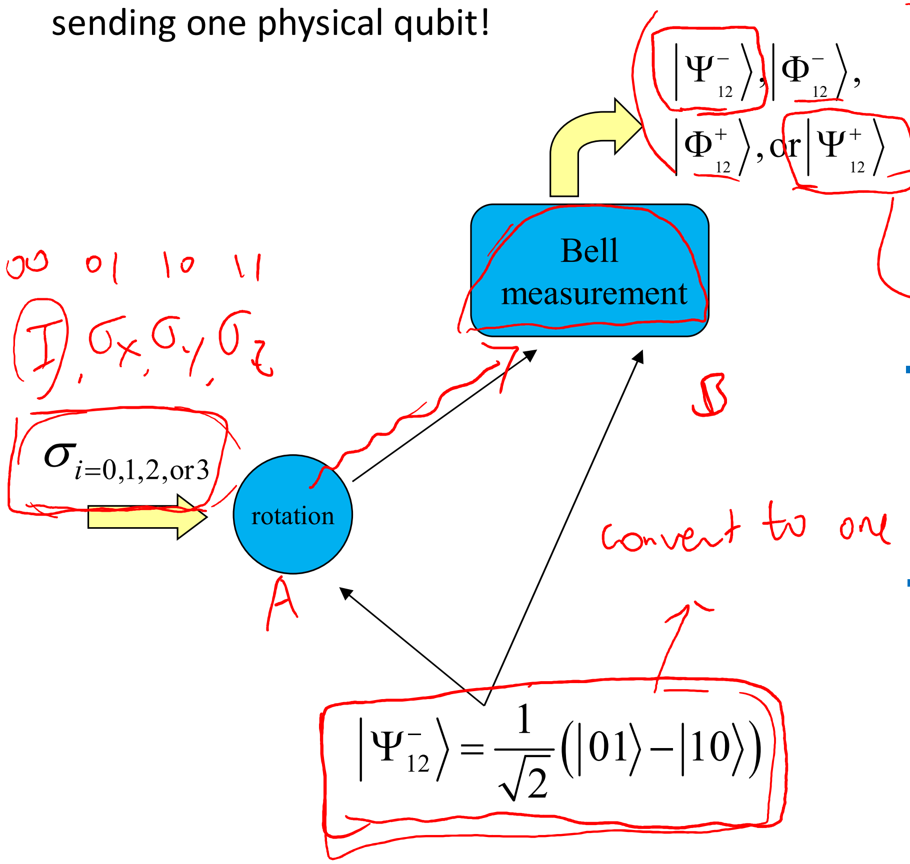
2. Finish Week 2 (From foundation to science-fiction teleportation)
3. Begin Week 3 material (Information is Physical)

Superdense coding

purpose : send classical info via sending a qubit

With shared entanglement, Alice can send two bits of classical message to Bob by sending one physical qubit!

⇒ communicate 2 bits of classical via a quantum bit



- Encode info. by certain operation *i*
- Decode it by identifying outcome of orthogonal measurements

convert to one of four Bell state

- Can send 2 bits of info. by consuming 1 singlet state

Key property

$$|\Phi^{\pm}\rangle \equiv \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

AB

$$|\Psi^{\pm}\rangle \equiv \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

$\sigma_x \otimes \mathbb{1} \longrightarrow (|10\rangle + |01\rangle) \Rightarrow \Psi^+$

$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$\sigma_z \otimes \mathbb{1} \downarrow (|00\rangle - |11\rangle) \Rightarrow |\Phi^-\rangle$

$\sigma_y \otimes \mathbb{1} \sim \sigma_x \sigma_z \otimes \mathbb{1} \longrightarrow (|01\rangle - |10\rangle)$

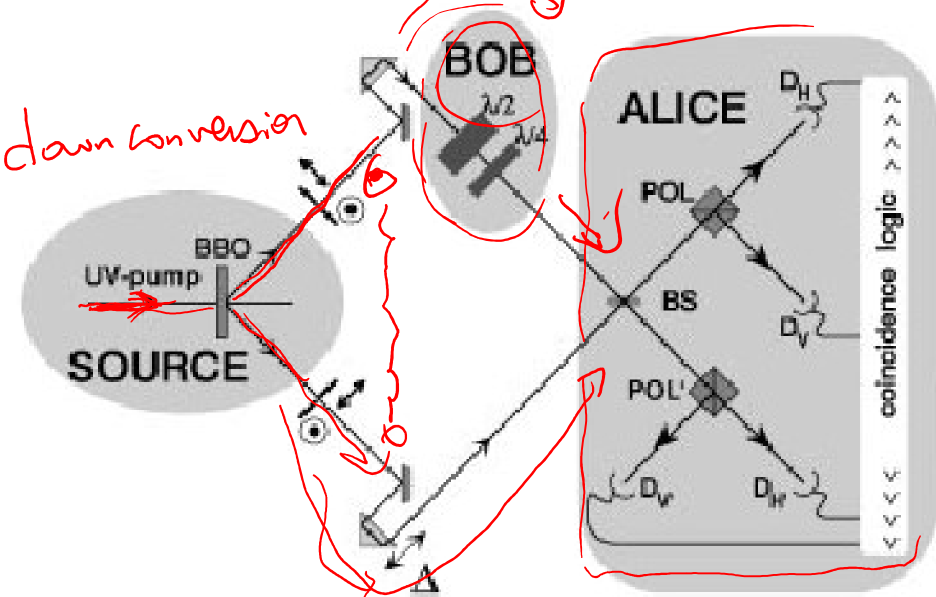
- The four Bell states can be interconverted to one another via local operation by either A or B

Superdense coding experiment

rotation to encode message

[Mattle et al. '96]

down conversion



Bell measurement

This was verified.

Simplest setup using linear-optic has success rate 50%

in practice 2 messages

3 message

Could use more degree of freedom

4 messages

FYI: Superdense teleportation*

Alice wants to send the state (with phases ϕ's supplied by Charlie) to Bob

– remote state preparation

$$\rightarrow |\psi\rangle_1 = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1} e^{i\phi_j}|j\rangle, \quad \text{with } \phi_0 = 0$$

$\rightarrow$ d-level

$1|0\rangle + e^{i\phi_1}|1\rangle + e^{i\phi_2}|2\rangle + \cdots$

A B

φ 0

Charlie

phases

The shared entangled state between Alice and Bob is $|\Phi\rangle_{12} = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}|j,j\rangle$

qudit $|\Phi^+\rangle$

Alice

Charlie applies the phase shift $U_\phi = \sum_{j=0}^{d-1} e^{i\phi_j}|j\rangle\langle j|$ to particle 1 (of Alice) and measure it in the basis defined by

$e^{i\phi_j}|jj\rangle$

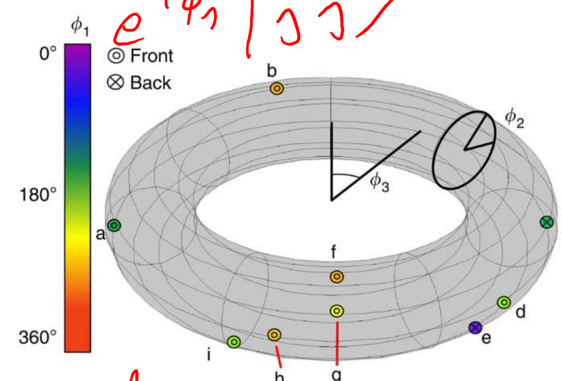
$|\Phi\rangle = \frac{1}{\sqrt{d}}\Sigma$

Fourier basis $\leftarrow$

$$|\tilde{k}\rangle = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1} e^{i2\pi jk/d}|j\rangle$$

perform meas $\rightarrow$

$$\langle\tilde{k}|_1 U_\phi \otimes I|\Phi\rangle_{12} = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1} e^{i\phi_j}\langle\tilde{k}|_1 \cdot |j,j\rangle = \frac{1}{d}\sum_j e^{i\phi_j - i2\pi jk/d}|j\rangle$$



Alice informs Bob of outcome k, and Bob applies $V_k = \sum_{j=0}^{d-1} e^{i2\pi jk/d}|j\rangle\langle j|$

cancel additional phase

to his particle to recover ψ

Graham, Bernstein, Wei, Junge, Kwiat, Nature Communications 6, 7185 (2015)

Week 3: Information is physical---Physical systems for quantum information processing: Superconducting qubits, solid-state spin qubits, photons, trapped ions, and topological qubits (p-wave superconductors, fractional quantum Hall systems, topological insulators, etc.)

DiVincenzo's criteria

1. A scalable physical system with well-characterized qubits

ΔE $|0\rangle$ $|1\rangle$ many qubits

(5)   (2)

2. The ability to initialize the state of the qubits to a simple fiducial state, such as 00…0

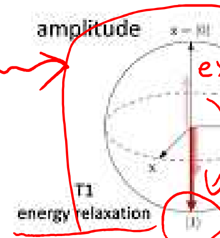
State preparation

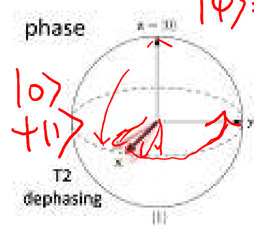
3. Long relevant decoherence times (relaxation T1, dephasing T2), much longer than the gate operation time

decohere

excited

want T_1, T_2 long





$|\psi\rangle = a|0\rangle + b|1\rangle$

cross term

$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix} |a|^2 & 1 \cdot ab^* \\ 1 \cdot a^*b & |b|^2 \end{pmatrix}$$

< 1

4. A "universal" set of quantum gates: e.g. Hadamard gate, T gate and CNOT gates (discussed more in later lectures)

5. A qubit-specific measurement capability

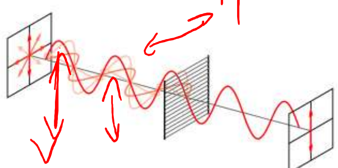
Physical qubits---Photons & light

Advantage: clean, low decoherence
Disadvantage: hard to entangle, loss of photons can be an issue

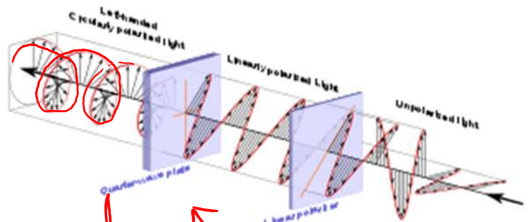
Bloch

1. **Polarization** (direction of Electric field): Horizonal 0, Vertical 1 (Z-basis); Diagonal , Antidiagonal (X-basis); Right circular, Left circular (Y-basis)

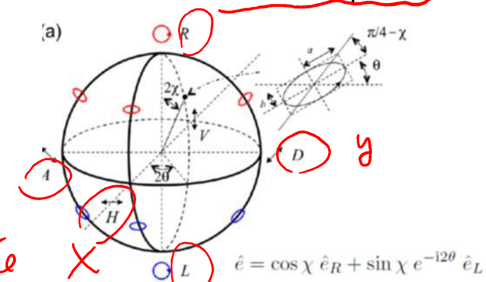
Their qubit sphere is also called Poincare sphere



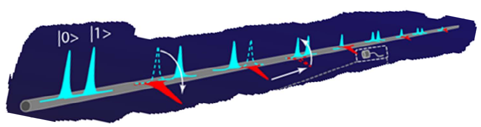
polarizer vertical

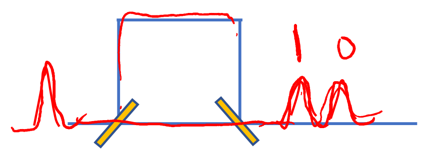


quarter waveplate



2. **Time bins** (pulse position): use a Mach-Zehnder interferometer to split the pulse





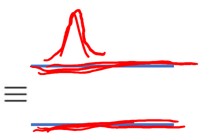
1 0

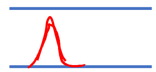
time is important

Physical qubits---Photons & light (cont'd)

(modes)

3. **Dual-rail encoding**:

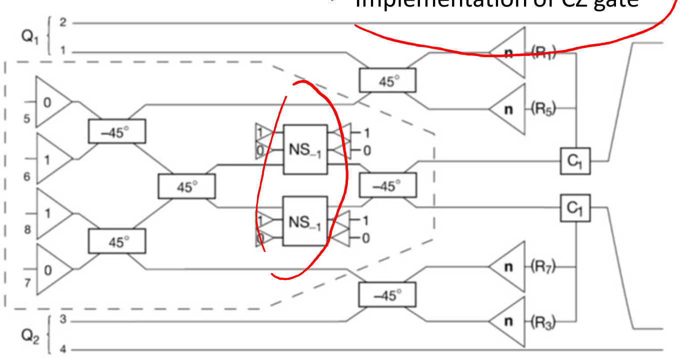
$|0\rangle \equiv$  $\hat{a}_1^\dagger|\text{vacuum}\rangle$

$|1\rangle \equiv$  $\hat{a}_2^\dagger|\text{vacuum}\rangle$

Used in Knill-Laflamme-Milburn scheme of linear-optic quantum computation [Nature 409, 46, 2001]

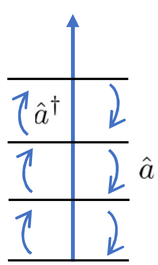
photons usually don't interact

Implementation of CZ gate



4. **Continuous-variable**: e.g. coherent state $\boxed{\hat{a}|\alpha\rangle = |\alpha\rangle}$

$\hat{a} = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{p})$



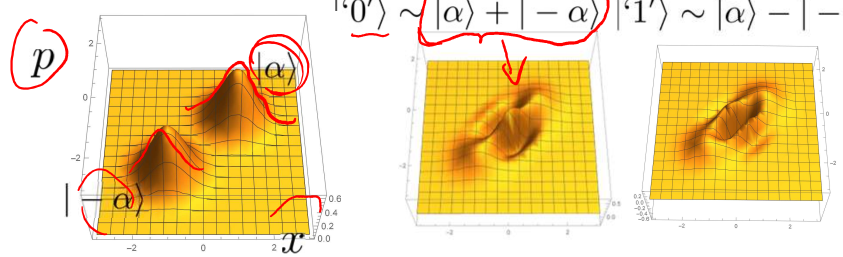
lower

$$\hat{a} = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{p})$$

raising

$$\hat{a}^\dagger = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{p})$$

$|‘0’\rangle \sim |\alpha\rangle + |-\alpha\rangle \quad |‘1’\rangle \sim |\alpha\rangle - |-\alpha\rangle$



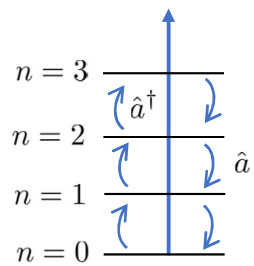
Photonic (bosonic) creation and annihilation

$$|1\,\text{photon}\rangle = \hat{a}^\dagger|\text{vacuum}\rangle$$

$$\sqrt{2}|2\,\text{photon}\rangle = (\hat{a}^\dagger)^2|\text{vacuum}\rangle$$

$$\sqrt{n!}|\text{n photon}\rangle = (\hat{a}^\dagger)^n|\text{vacuum}\rangle$$

$$\boxed{\hat{a}^\dagger|\text{n photon}\rangle = \sqrt{n+1}|\text{n}+1\,\text{photon}\rangle}$$



$$\hat{a}|1\,\text{photon}\rangle = |\text{vacuum}\rangle$$

$$\hat{a}^2|2\,\text{photon}\rangle = \sqrt{2}|\text{vacuum}\rangle$$

$$\hat{a}|\text{n photon}\rangle = \sqrt{n}|\text{n}-1\,\text{photon}\rangle$$

Hermitian conjugate

$$\hat{a}^\dagger = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & \cdots \\ 1 & 0 & 0 & \cdots & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \cdots & 0 & \cdots \\ 0 & 0 & \sqrt{3} & \ddots & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots \\ \vdots & 0 & 0 & \vdots & \sqrt{n} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\hat{a} = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & \cdots & \cdots \\ 0 & 0 & \sqrt{2} & 0 & \cdots & 0 & \cdots \\ 0 & 0 & 0 & \sqrt{3} & \vdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots \\ \vdots & 0 & 0 & 0 & \vdots & \sqrt{n} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[\hat{a}, \hat{a}^\dagger] \equiv \hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = I$$

$$\hat{a}|\alpha\rangle = |\alpha\rangle$$

Coherent state: $|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$

Exercise: coherent state

(a) Verify that the coherent state $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$

satisfy $\hat{a}|\alpha\rangle = |\alpha\rangle$ by using $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$

(b) Verify that this coherent state is properly normalized: $\langle\alpha|\alpha\rangle = 1$

(c) Show that the logical '0' and '1' states have even and odd number of photons, respectively, and hence they are orthogonal:

$$|`0'\rangle \sim |\alpha\rangle + |-\alpha\rangle \quad |`1'\rangle \sim |\alpha\rangle - |-\alpha\rangle$$

Physical qubits---Spins

❑ Spins: electron spins, diamond NV center, quantum dots, etc.

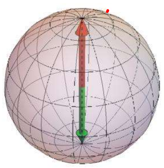
In general, spin angular momentum operators are associated with generators of rotation (ħ≡1) :

$$(-i)S_\alpha = \frac{d}{d\theta}\Big|_{\theta\to 0} R_\alpha(\theta) \qquad [S_x, S_y] = iS_z,\ [S_y, S_z] = iS_x,\ \text{etc.}$$

➢ Spin-1/2 particles have two states up (S_z=+1/2) and down (S_z=-1/2), and spin operators are related to Pauli matrices:

$S_m = \frac{1}{2}$ $\quad S_x = \frac{\sigma_x}{2}$ $\quad S_y = \frac{\sigma_y}{2}$ $\quad S_z = \frac{\sigma_z}{2}$

$$\vec{S}_\alpha = \frac{1}{2}\vec{\sigma}_\alpha = \frac{1}{2}(X, Y, Z)$$

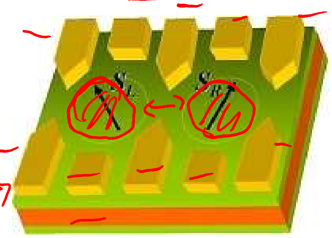


Advantage: spin-1/2 is precise 2-level system; controllable by magnetic field
Disadvantage: solid-state environment is noisy; short coherence time

Physical qubits---Spins (cont'd)

- Quantum dots: electrically confined dots that host an effective electron on each dot [Loss & DiVincenzo, Phys. Rev. A 57, 120 (1998)]

semiconductor



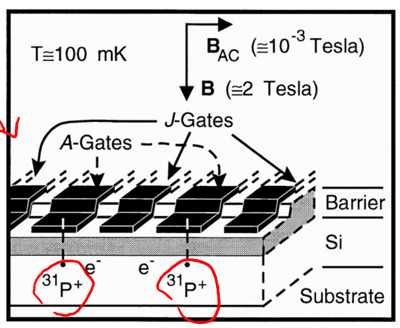
electrodes (gate)

- The two electrons interact via Heisenberg coupling, whose strength can be tuned:

$$\hat{H} = J\vec{S}_L \cdot \vec{S}_R = J\left[(S_L)_x(S_R)_x + (S_L)_y(S_R)_y + (S_L)_z(S_R)_z\right]$$

- Phosphorus donors on pure silicon (nuclear spin) [Kane, Nature 393, 133 (1998)]

long coherence nuclear spin



- Both systems have the advantage of integrating with current silicon technology

[There is national research effort of Australia on this kind of qubit]

Physical qubits---Spins (cont'd)

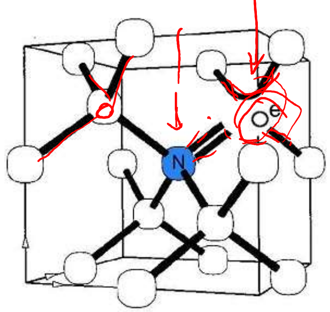
- Nitrogen-vacancy (NV) center in diamond:
  e.g. the negatively charged state N-V⁻ → electron spins (ground state with S=1) can be manipulated by electric field and magnetic field

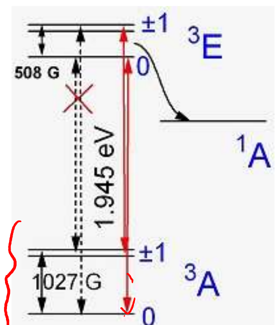
singlet triplet

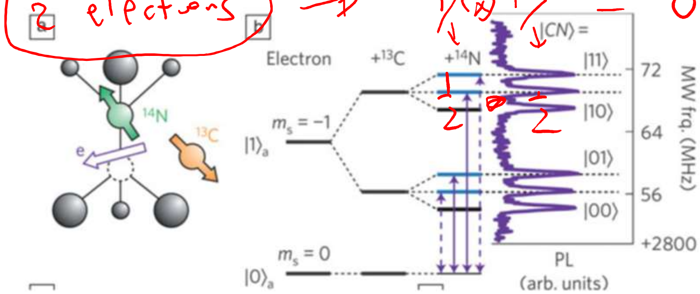
miss Carbon atom

2 electrons ⇒ $\uparrow \otimes \uparrow = 0 \oplus 1$

$\boxed{S=1}$ → 3 sub levels $m_s = 0, \pm 1$



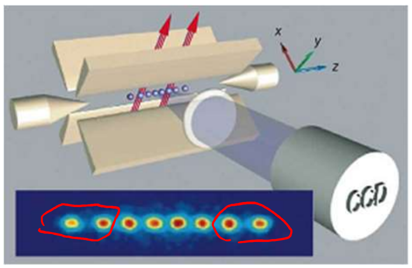




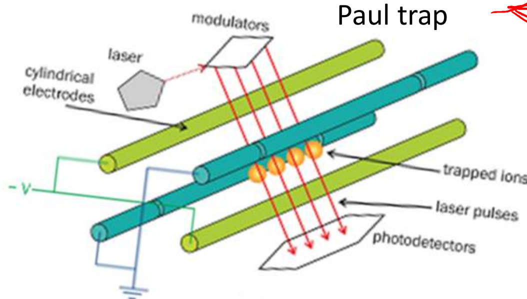
- Coherence time is micro- to milliseconds (can be made longer by "dynamical decoupling")
- Can also use a NV center and nearby ^{13}C nuclear spin for quantum operation

Physical qubits---Trapped ions

Trapped Ions:

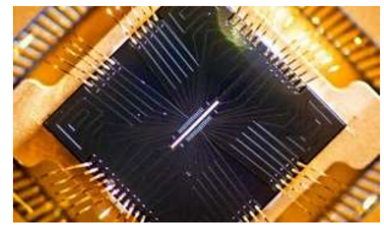


Paul trap



$$\Phi_{\rm dc}(x,y,z) = \kappa U_0[z^2 - (x^2+y^2)]/2$$

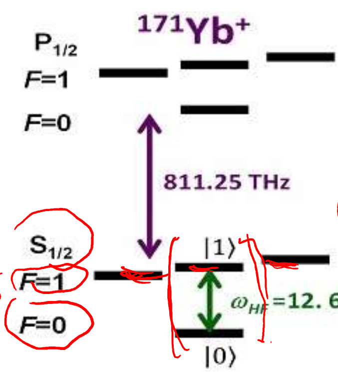
$$\Phi_{\rm rf}(x,y) = (V_0 \cos\Omega_T t + U_r)(1 + (x^2-y^2)/R^2)/2$$



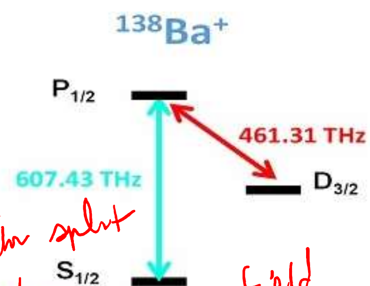
Chip and trap (Sandia NL)

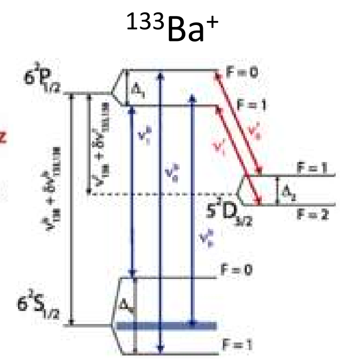
Use selective hyperfine levels as qubit basis state

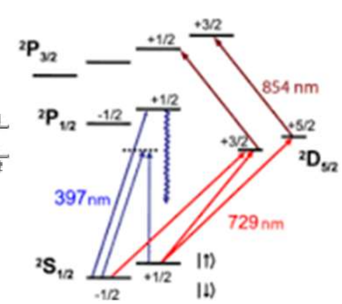
- $^{171}Yb^+$ (used by IonQ, Monroe's group); $^{40}Ca^+$ (Blatt's group); $^{9}Be^+$ (Wineland's goup)



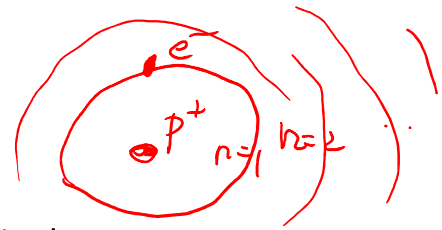
Can be further split by magnetic field







Electronic levels, Fine and Hyperfine structures (of hydrogen)*



- Most of you know energy levels of a hydrogen atom $E_n = -\frac{13.6\text{eV}}{n^2}$

  and some orbitals, 1s, 2s, 2p, 3s, 3p, 3d, etc. (where n=1,2,3.. Is the principal quantum number)

$^{2S+1}L_J$: L orbital angular momemntum, S spin angular momentum, J total (electronic) angular momentum

S (l=0) . P (l=1) D (l=2)

Fine structure

degenerate

$-\frac{13.6\text{eV}}{4}$ $n=2$ 2s, 2p

$2P_{\frac{3}{2}}$

$2S_{\frac{1}{2}}$ $2P_{\frac{1}{2}}$

↑ larger j

j = l + s

$$E_{nj} = -\frac{13.6\text{eV}}{n^2}\left[1 + \frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right)\right]$$

Fine structure: due to Relativistic effect + Spin-orbit interaction

-13.6eV $n=1$ 1s

$$\vec{J} = \vec{L} + \vec{S}$$

orbital, spin

Hyperfine structure

due to interaction of electron and nuclear spins

$$\vec{F} = \vec{J} + \vec{I}$$

nuclear spin

$S_{\frac{1}{2}}$

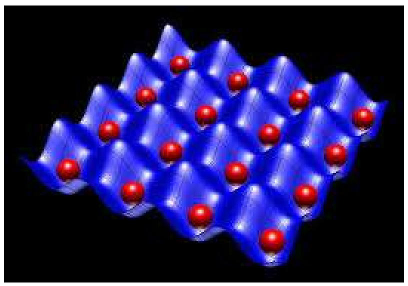
triplet $F = 1$

$\nu = \frac{\Delta E}{h} = 1420\text{MHz}$ 21 cm

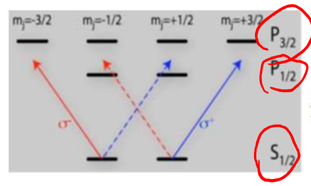
singlet $F = 0$

Trapped neural atoms

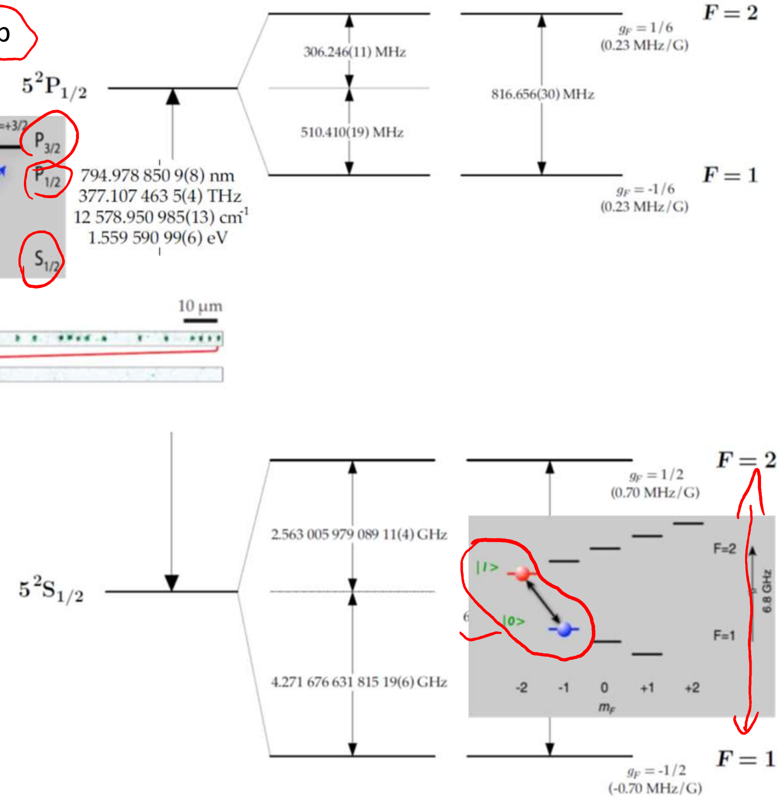
- Optical Lattice

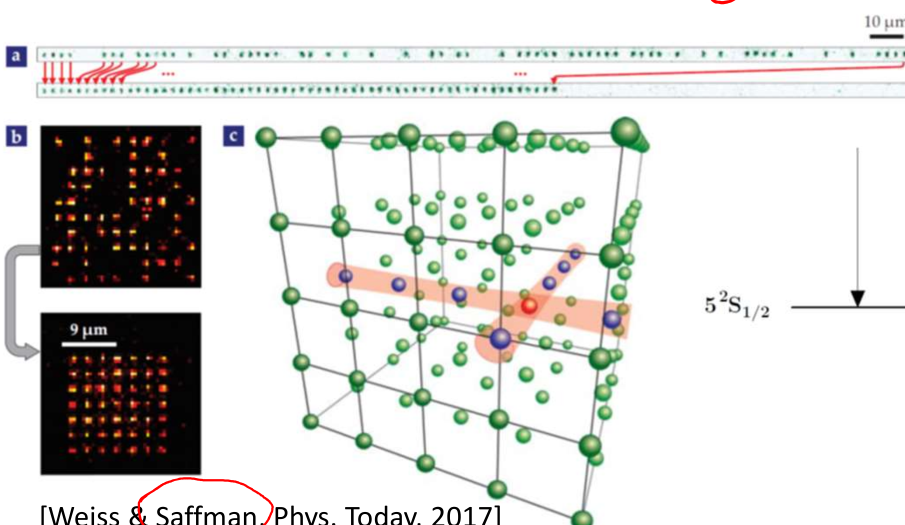


Fine levels of ^{87} Rb



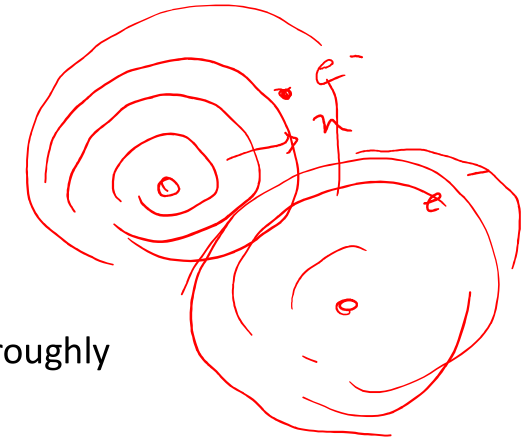
Hyperfine levels of ^{87} Rb



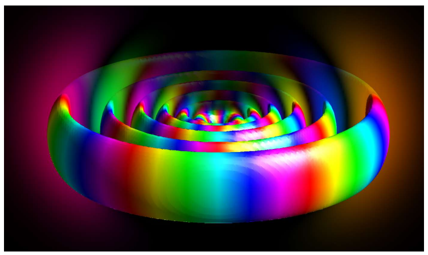


[Weiss & Saffman, Phys. Today, 2017]

Trapped neural atoms (cont'd)



- Rydberg atoms (high n number)



[n=12 state, from Wikipedia]

- For hydrogen-like atoms, radius is roughly

$$r_n \sim n^2 a_{\mathrm{Bohr}}/Z$$

- Two hyperfine ground states |0> and |1> and use a Rydberg state |r> to construct a controlled gate (Rydberg blockade)