PHY682 Special Topics in Solid-State Physics: Quantum Information Science

Lecture time: 2:40-4:00PM Monday & Wednesday

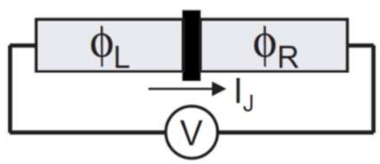
Last time 9/9: DiVincenzo's criteria; photons and continuous variables; spins (electrons and nuclear); trapped ions and atoms (hyperfine states)

Today 9/14:

1. Finish Week 3 material (Information is Physical): Superconducting qubits & topological qubits
2. Begin Week 4 material: quantum gates [How to use Qiskit]

Physical qubits---Superconducting qubits

- Superconducting qubits: (a) Phase; (b) Flux; (c) Charge; (d) Transmon/Xmon
  - Crucial ingredient: Josephson junction → nonlinear inductance



current

$$\boxed{I_J = I_0 \sin(\phi_L - \phi_R) = I_0 \sin\delta \\ V = \frac{\Phi_0}{2\pi}\frac{d\delta}{dt}, \quad \Phi_0 \equiv \frac{h}{2e}}$$

$\frac{d\delta}{dt} \sim V$

current flows

$e^- \; e^-$

SC SC

ϕ_L

superconducting phase

oxides

$e^{i\phi_R}$

$e^{i\phi_L}$

voltage

V

wire

- Inductance: $V = L\frac{dI}{dt}$ current

usually does not depend on current

$$V = L_J \frac{dI_J}{dt} = L_J I_0 \cos\delta \cdot V \frac{2\pi}{\Phi_0}$$

$\frac{d\delta}{dt}$

$$L_J = \frac{\Phi_0}{2\pi I_0 \cos\delta} = \pm \frac{\Phi_0}{2\pi\sqrt{I_0^2 - I_J^2}}$$

current dependence

- Energy stored in junction:

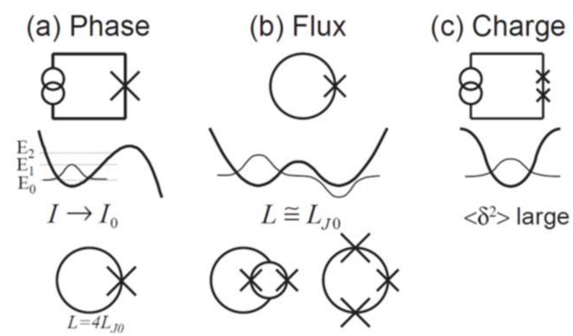
Josephson energy

E_J

$$U = \int V I_J \, dt = -\frac{\Phi_0 I}{2\pi}\cos\delta$$

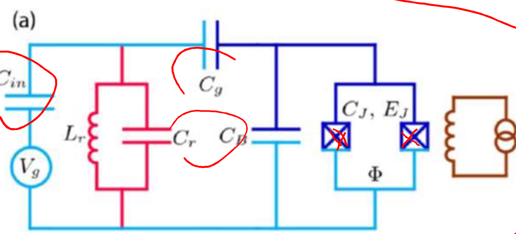
starting point for energy consideration

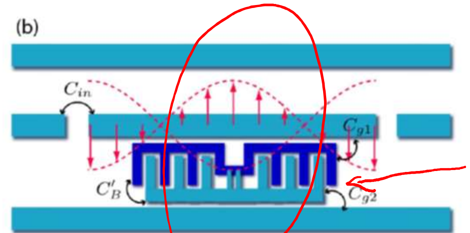
I



Physical qubits---Superconducting qubits

❑ Transmon qubit [Koch et al. PRA 76, 042319 (2007)]





$\hat{n}$ dark blue

$\updownarrow \delta$

circuit diagram

design of materials

➢ Essential 'Hamiltonian':

(energy)

$$\hat{H}(n_g) = 4E_c(\hat{n} - n_g)^2 - E_J \cos\hat{\delta}$$

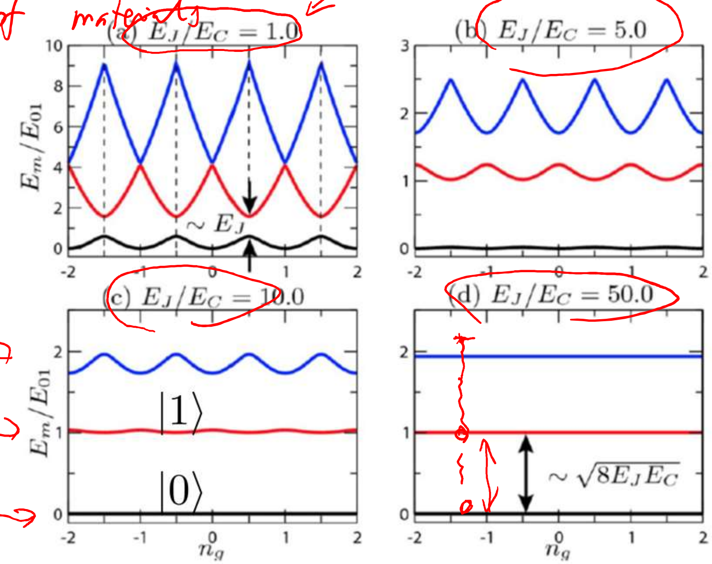
$\hat{n}$: number of Cooper pairs

n_g : charge offset

$\hat{\delta}$: phase between superconductors

E_c : charging energy

E_J : Josephson energy



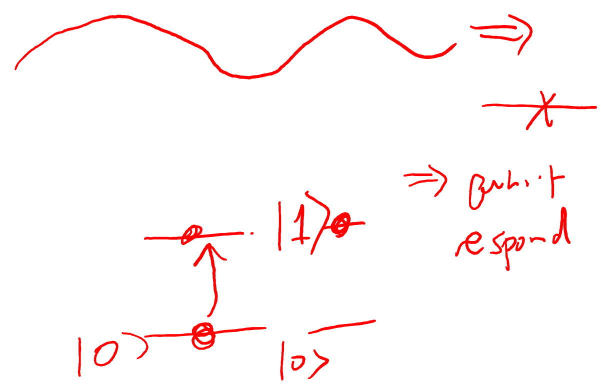
• E_J big cos term dominates

• E_C dominant

not equal level

Useful references on SC qubits and how to control them via Qiskit Pulse

Qiskit Pulse: Programming Quantum Computers Through the Cloud with Pulses, Thomas Alexander, Naoki Kanazawa, Daniel J. Egger, Lauren Capelluto, Christopher J. Wood, Ali Javadi-Abhari, David McKay, arxiv:2004.06755



First-principles analysis of cross-resonance gate operation, Moein Malekakhlagh, Easwar Magesan, David C. McKay, arxiv:2005.00133

Fermions, bosons and anyons

statistics

- Fermions, such as electrons, cannot occupy the same state and their wavefunction gives a minus sign under particle exchange

anti commute

$$\Psi_F(x_1,\ldots,x_j,\ldots,x_k,\ldots) = -\Psi_F(x_1,\ldots,x_k,\ldots,x_j,\ldots)$$

annihilator
creation

$$\{\hat{c}_i,\hat{c}_j\} = \{\hat{c}_i^\dagger,\hat{c}_j^\dagger\} = 0 \quad \{\hat{c}_i,\hat{c}_j^\dagger\} = \delta_{ij}$$

$$c_i c_i^{+} + \hat{c}_i^{+} \hat{c}_i = \mathbb{1}$$

- Bosons, such as photons, prefer to occupy the same state and their wavefunction is the same under particle exchange

$$\Psi_B(x_1,\ldots,x_j,\ldots,x_k,\ldots) \overset{(+)}{=} \Psi_B(x_1,\ldots,x_k,\ldots,x_j,\ldots)$$

$$[\hat{b}_i,\hat{b}_j] = [\hat{b}_i^\dagger,\hat{b}_j^\dagger] = 0 \quad [\hat{b}_i,\hat{b}_j^\dagger] = \delta_{ij}$$

$$\hat{b}_i \hat{b}_i^{+} - \hat{b}_i^{+} \hat{b}_i = \mathbb{1}$$

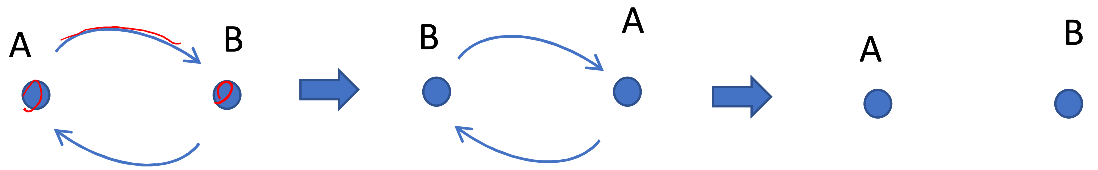
Commutator $\rightarrow$ $\hat{b}_i \hat{b}_j - \hat{b}_j \hat{b}_i = 0$

- Anyons are exotic particles and their wavefunction gives arbitrary phase under exchange [usually in two dimensions]

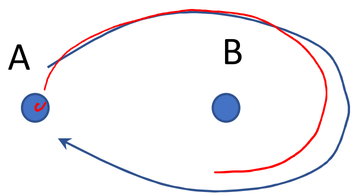
$$\psi_A(x_j \cdots x_i) = e^{i\theta}\,\psi_A(x_i \cdots x_j) \quad \text{or matrix}$$

Two exchanges = Looping

Exchange:



Looping:



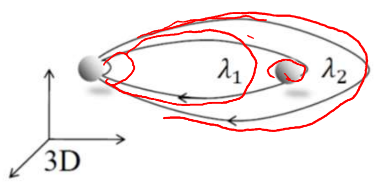
exchange

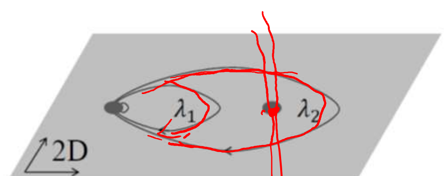
phase

$$\hat{P}_{ij}\Psi(x_1,\ldots,x_j,\ldots,x_k,\ldots) = e^{i\theta}\Psi(x_1,\ldots,x_k,\ldots,x_j,\ldots)$$

$$\hat{P}_{ij}^2 \Psi = \Psi$$

- 2D is different from 3D:





In 2D: the loop cannot be continuously shrunk to a point

$$\hat{P}_{ij}^2 \Psi = \Psi$$

strictly does not hold in 2D

$$\Rightarrow e^{i\theta} = \pm 1 \text{ boson or fermion}$$

➔ No constraint on particle statistics

can have exotic statistics

Exercise: bosons and fermions

(a) For two-mode bosonic operators and state, calculate

$$(a_2^\dagger)^2 a_1^\dagger |0_1, 0_2\rangle =?$$

(b) For single-mode fermionic operators and state,

$$\hat{c}|0\rangle = 0,\ \hat{c}^\dagger|0\rangle = |1\rangle,\ \hat{c}|1\rangle = |0\rangle,\ \hat{c}^\dagger|1\rangle = 0$$

Consider the following two-mode state:

$$|\psi\rangle \equiv c_2^\dagger c_1^\dagger |0_1, 0_2\rangle \equiv |1_1, 1_2\rangle$$

Compare the two following states by direct calculation: [you may need $\hat{c}\hat{c}^\dagger = I - \hat{c}^\dagger \hat{c}$]

$$c_2 c_1 |\psi\rangle \qquad c_1 c_2 |\psi\rangle$$

Do you see a sign difference?

Physical qubits---Topological systems

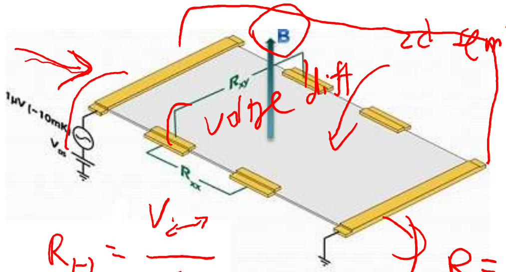
- Intrinsic topological phases harbor exotic particles called “anyons” and their braiding (braid group) gives rise to quantum gates. More on these in later lectures.

Hard to move anyons

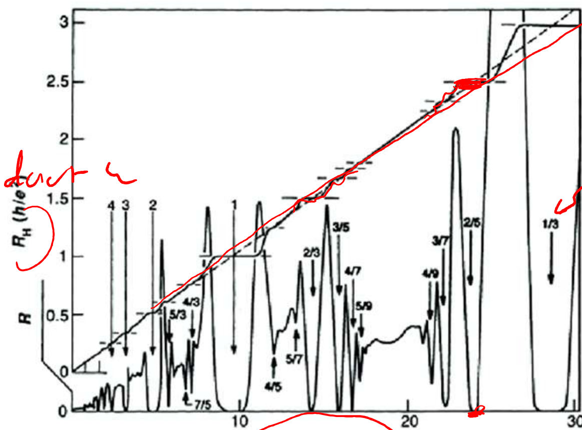
- ❑ Fractional Quantum Hall system

2d semiconductor

voltage drift



$R_H = \frac{V_{\leftrightarrow}}{I}$ $R = \frac{V}{I}$



anyons emerge from these fractional Hall values

$R \to 0$ dissipationless

R_H : flat

- ❑ Quantum spin liquids

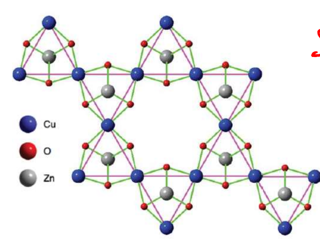
frustrated magnets

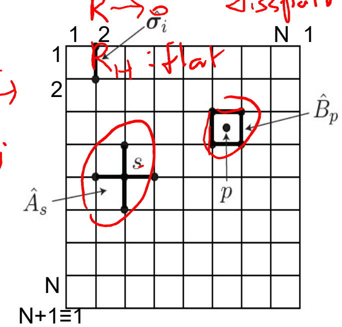
  - Heisenberg spin model on kagome lattice

$\vec{S}_i \cdot \vec{S}_j$

  - Kitaev’s toric code model

$ZnCu_3(OH)_6Cl_2$ herbertsmithite





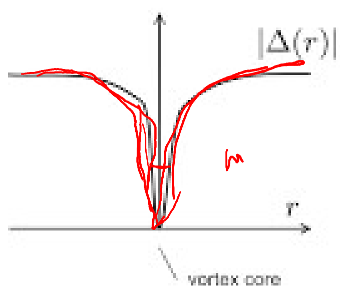
Toric code

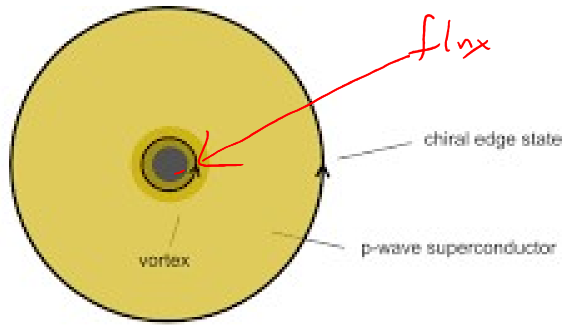
$$H = -\sum_{s} \sigma_x \otimes \sigma_x \otimes \sigma_x \otimes \sigma_x - \sum_{p} \sigma_z \otimes \sigma_z \otimes \sigma_z \otimes \sigma_z$$

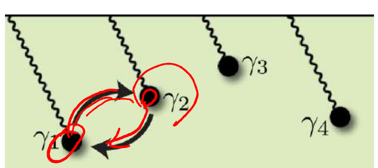
Physical qubits---Topological systems

- 2D p+ip superconductor → Majorana fermion (see also next slide) [Ivanov, PRL 2001]

state







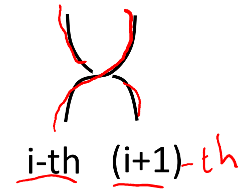
- Effect of exchanging (or braiding) two vortices

$$\gamma_1 \to \gamma_2$$
$$\gamma_2 \to -\gamma_1$$

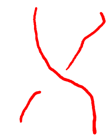
additional phase ⇒ used for quantum gates

Braid group

- ❑ Use T_i to represent braiding of i-th and (i+1)-th threads:

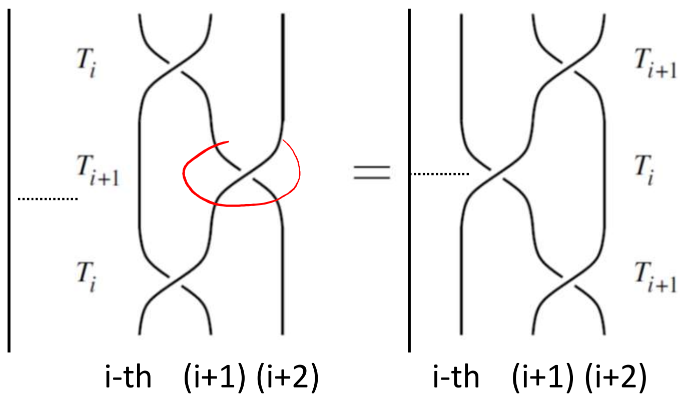


T_i^{-1}

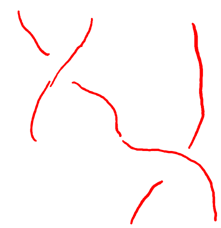


- ❑ They form a group and there is some constraint:

$$T_i T_{i+1} T_i = T_{i+1} T_i T_{i+1}$$



identical braiding



- ❑ The goal is to find anyons whose braid group forms universal gates

➔ Fibonacci anyons are universal (☺), but Ising anyons are not (☹) → Some gates cannot be realized

Majorana fermions (Majorana Zero Mode)

p-wave

can only "square" half of a fermion

- Usual fermions (aka Dirac fermions) satisfy

$$\{\hat{c}_i, \hat{c}_j\} = \{\hat{c}_i^\dagger, \hat{c}_j^\dagger\} = 0 \quad \hat{c}_i^2 = 0 = (\hat{c}_i^\dagger)^2 \quad \{\hat{c}_i, \hat{c}_j^\dagger\} = \delta_{ij}$$

$c_i c_i + c_i c_i = 0$

- Majorana fermions:

real part & imaginary of an ordinary fermion

$\gamma_B^\dagger = c^\dagger + c \Rightarrow$ $\hat{\gamma}_B \equiv \hat{c} + \hat{c}^\dagger$

$\gamma_A^\dagger = \frac{c^\dagger}{-i} - \frac{c}{-i} =$ $\hat{\gamma}_A \equiv (\hat{c} - \hat{c}^\dagger)/(i)$

$$\hat{\gamma}_A^2 = I = \hat{\gamma}_B^2 \qquad \hat{\gamma}_B^\dagger = \hat{\gamma}_B, \; \hat{\gamma}_A^\dagger = \hat{\gamma}_A$$

$\gamma_B^2 = (c + c^\dagger)^2 = \underset{0}{cc} + \underbrace{cc^\dagger + c^\dagger c}_{=1} + \underset{0}{c^\dagger c^\dagger}$

- They are like 'halves' of a fermion

- Can arise from e.g. Kitaev's fermion chain (with p-wave pairing)

$$H = -\sum_{x=1}^{N-1} \hat{c}_x^\dagger \hat{c}_{x+1} + \hat{c}_x \hat{c}_{x+1} + h.c. = -i \sum_{x=1}^{N-1} \hat{\gamma}_{B,x} \hat{\gamma}_{A,x+1}$$

Cooper pair

can be used to encode a qubit

physical fermion site

$\gamma_{A,1} \cdot \gamma_{B,N}$ free

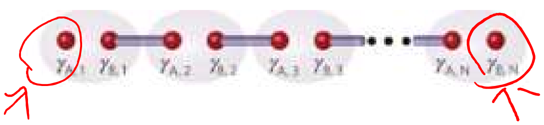
Exercise: proof of the Majorana picture

pairing

$$H = -\sum_{x=1}^{N-1}(\hat{c}_x^\dagger \hat{c}_{x+1} + \hat{c}_x \hat{c}_{x+1} + \hat{c}_{x+1}^\dagger \hat{c}_x + \hat{c}_{x+1}^\dagger \hat{c}_x^\dagger) = -i\sum_{x=1}^{N-1} \hat{\gamma}_{B,x}\hat{\gamma}_{A,x+1}$$

hopping

show this



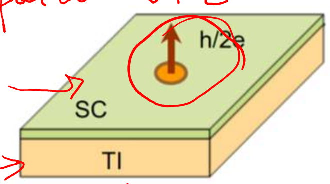
$$\hat{c}_x = \hat{\gamma}_{B,x} + i\hat{\gamma}_{A,x} \qquad \hat{c}_x^\dagger = \hat{\gamma}_{B,x} - i\hat{\gamma}_{A,x}$$

Hint: re-write the terms in the summand of c operators in a product form: (site x) times (site x+1)

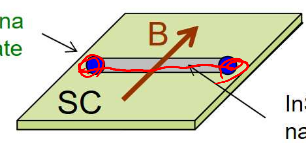
Braiding Majorana fermions

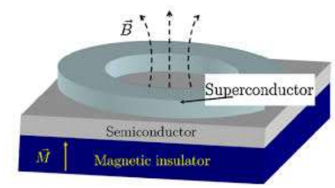
- ❑ Where to get Majorana fermions?

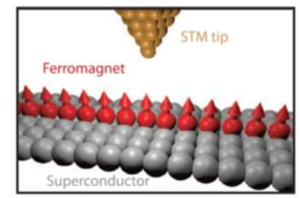
s-wave superconductor



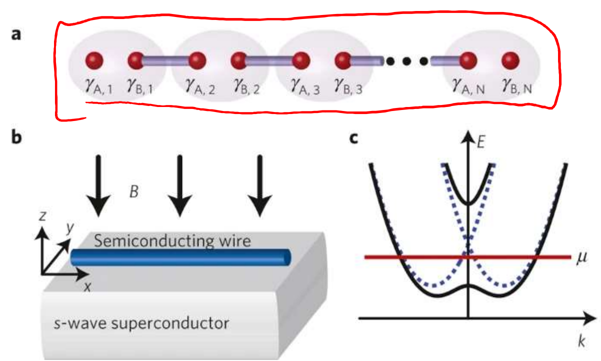
Topological Insulator

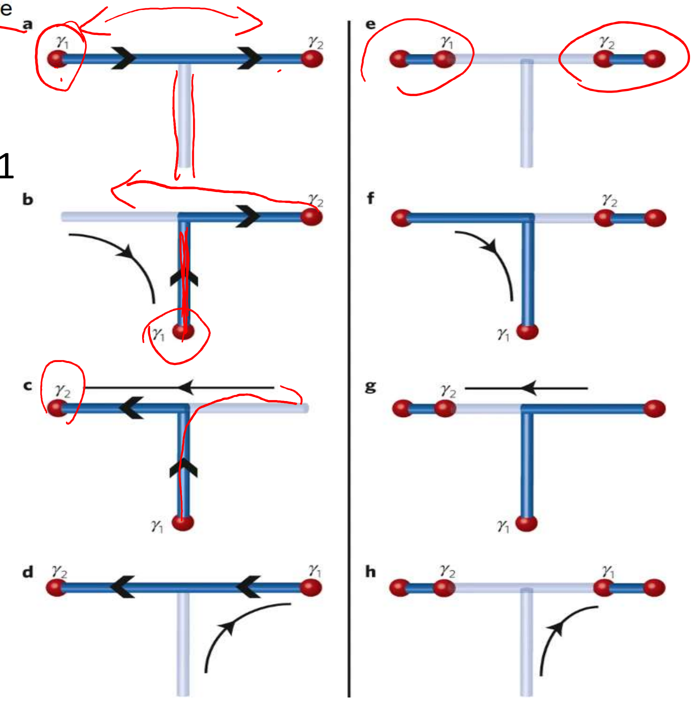




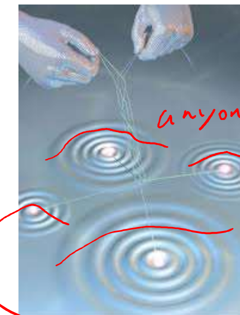


- ❑ Braiding: e.g. Alicea et al. Nature Physics 2011





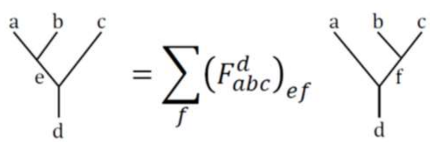
Why topological qubits?

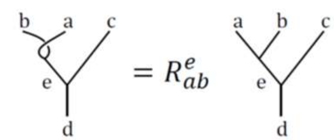


- Topological quantum computation is robust against noise (does not need active error corrections) [more later]

- Beautiful and elegant mathematics

$$a \times b = \sum_{c \in M} N_{ab}^{c}\, c$$





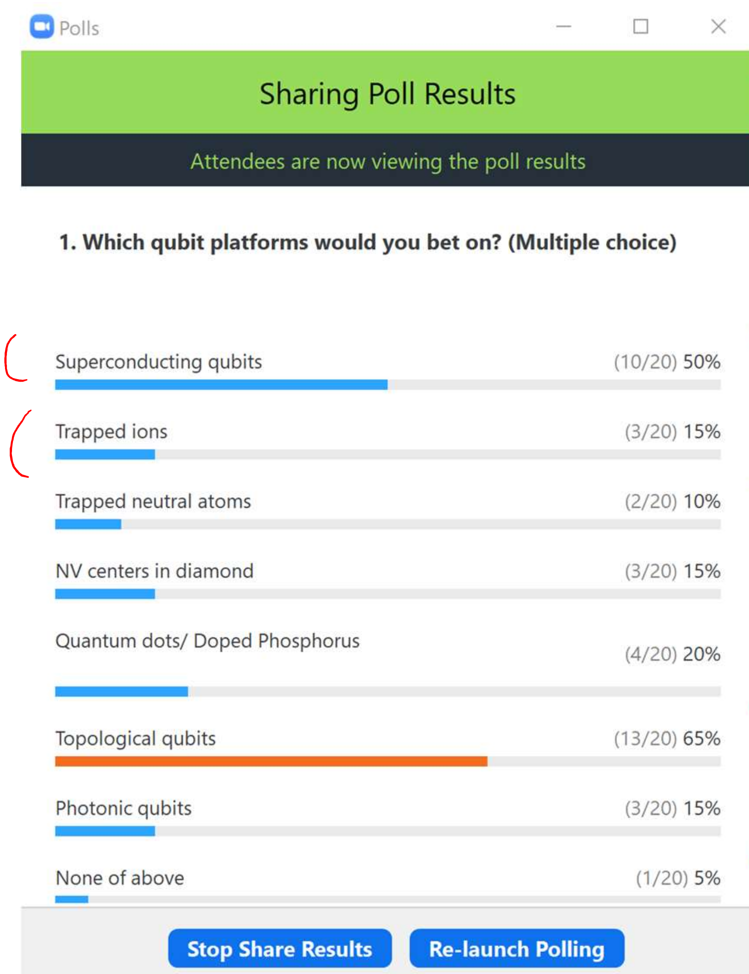
- Gates implemented via ‘braiding anyons’

- However, it is still very challenging to realize good topological qubits [Effort from academia and industry such as Microsoft (both theory and experiments)]

Do Poll

Which qubit platforms would you bet on?

(a) Superconducting qubits
(b) Trapped ions
(c) Trapped neutral atoms
(d) NV centers in diamond
(e) Quantum dots/ Doped Phosphorus
(f) Topological qubits
(g) Photonic qubits
(h) None of above



...in magnetism

...o ?

...tum free Hamiltonian

$\sum_i \hat{h}_i$

...nd state $|\psi\rangle$

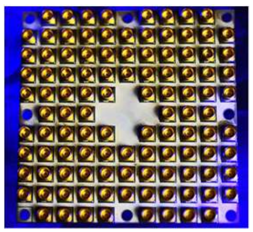
$\langle\psi|\hat{h}_i|\psi\rangle$ has

...est possible energy

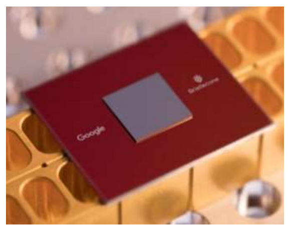
frustration free

Some existing quantum computers

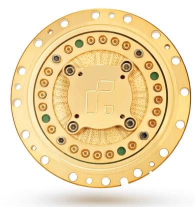
IBM 50-Qubit Q Computer



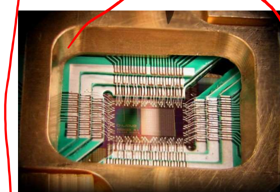
Intel 49-Qubit QC



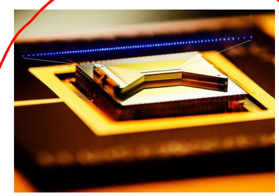
Google 72-Qubit QC



Rigetti 20-Qubit QC



D-Wave 2000-Qubit Annealer



IonQ 160-Qubit QC

non-superconducting one

→ More will appear (including other platforms)

much noisy

Physical systems for quantum simulations

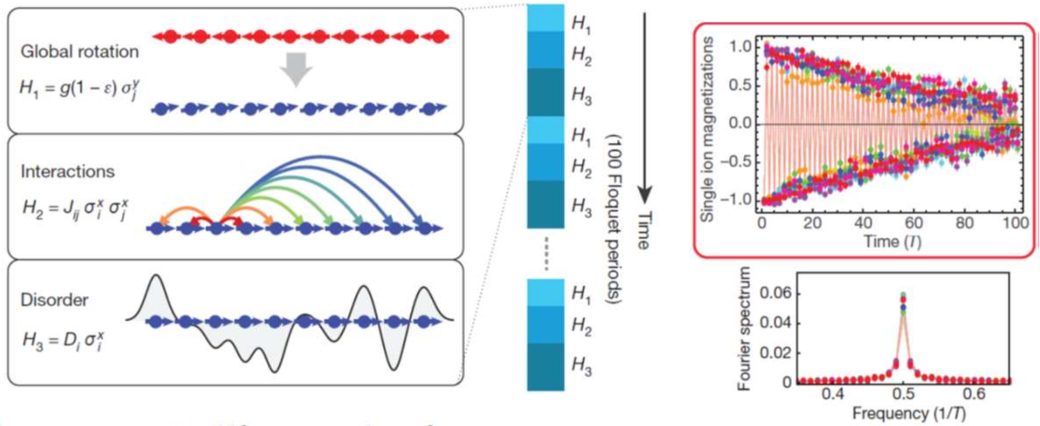
Observation of a discrete time crystal

J. Zhang[1], P. W. Hess[1], A. Kyprianidis[1], P. Becker[1], A. Lee[1], J. Smith[1], G. Pagano[1], I.-D. Potirniche[2], A. C. Potter[3], A. Vishwanath[2,4], N. Y. Yao[2] & C. Monroe[1,5]

doi:10.1038/nature21413

$$H=\begin{cases} H_1=g(1-\varepsilon)\sum_i \sigma_i^y & \text{time } t_1 \\ H_2=\sum_i J_{ij}\sigma_i^x\sigma_j^x & \text{time } t_2 \\ H_3=\sum_i D_i\sigma_i^x & \text{time } t_3 \end{cases}$$

10 Yb+ ions



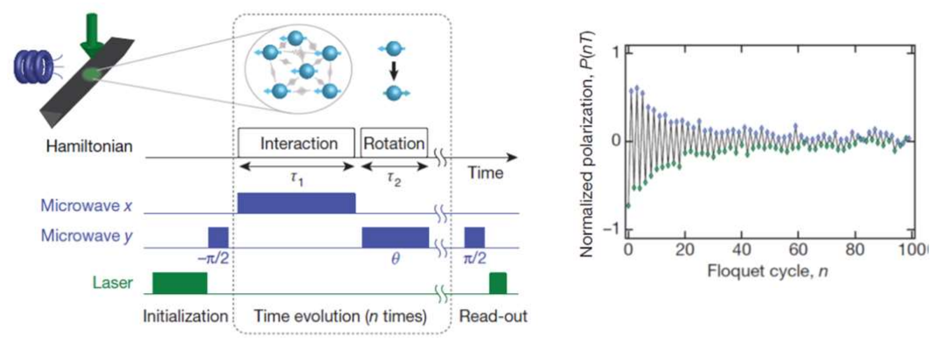
Observation of discrete time-crystalline order in a disordered dipolar many-body system

Soonwon Choi[1*], Joonhee Choi[1,2*], Renate Landig[1*], Georg Kucsko[1], Hengyun Zhou[1], Junichi Isoya[3], Fedor Jelezko[4], Shinobu Onoda[5], Hitoshi Sumiya[6], Vedika Khemani[1], Curt von Keyserlingk[7], Norman Y. Yao[8], Eugene Demler[1] & Mikhail D. Lukin[1]

doi:10.1038/nature21426

$$H(t)=\sum_i \Omega_x(t)S_i^x+\Omega_y(t)S_i^y+\Delta_i S_i^z + \sum_{ij}\left(J_{ij}/r_{ij}^3\right)\left(S_i^xS_j^x+S_i^yS_j^y-S_i^zS_j^z\right)$$

[system: high centration NV centers (45ppm)]



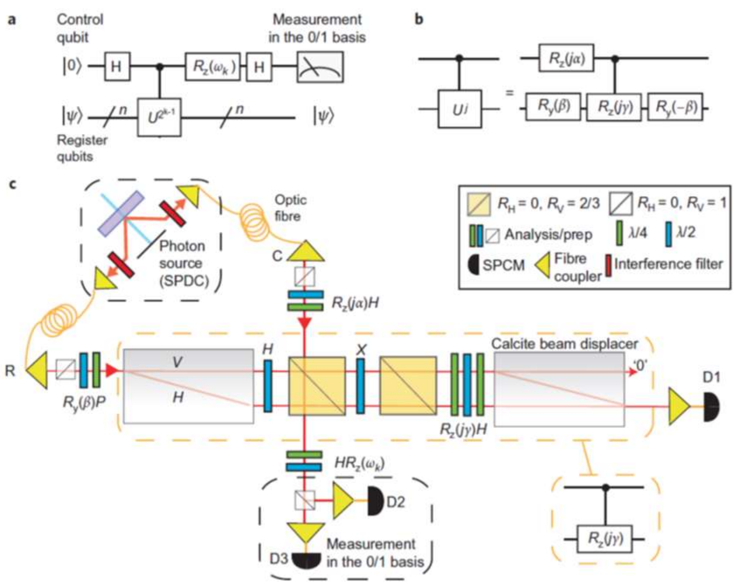
nature chemistry

ARTICLES

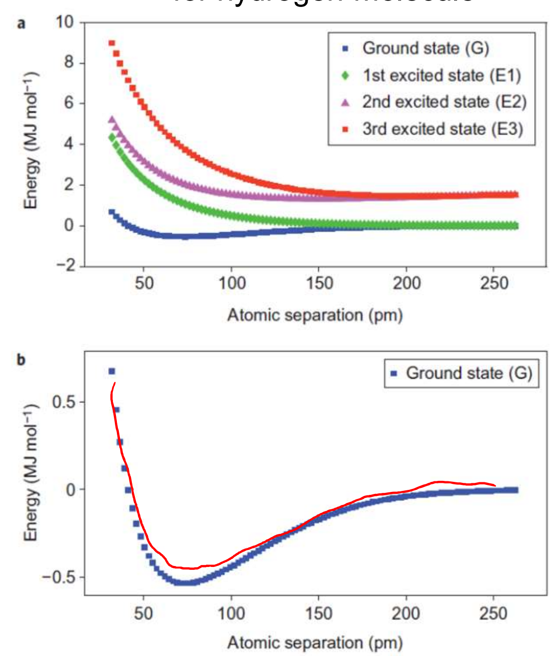
PUBLISHED ONLINE: 10 JANUARY 2010 | DOI: 10.1038/NCHEM.483

Towards quantum chemistry on a quantum computer

B. P. Lanyon[1,2*], J. D. Whitfield[4], G. G. Gillett[1,2], M. E. Goggin[1,5], M. P. Almeida[1,2], I. Kassal[4], J. D. Biamonte[4†], M. Mohseni[4†], B. J. Powell[1,3], M. Barbieri[1,2†], A. Aspuru-Guzik[4*] and A. G. White[1,2]



for hydrogen molecule



H H

separat

➔ Later in the course, we will use IBM quantum computers for such computation

Two more DiVincenzo criteria

Needed for quantum communication:

1. The ability to interconvert stationary and flying qubits

2. The ability to faithfully transmit flying qubits between specified locations