PHY682 Special Topics in Solid-State Physics: Quantum Information Science

Lecture time: 2:40-4:00PM Monday & Wednesday

Today 11/16:

1. Final presentation selection and presentation outline on Blackboard
2. Week 13's topics (quantum simulations and metrology)

Presentation topics

Group 1: “Entanglement-Based Machine Learning on a Quantum Computer”, PhysRevLett.114.110504 (2019)

Group 2: “Universal Blind Quantum Computation” (3 related references)

Group 3: “Can the ‘WaveFunctionCollapse’ algorithm run on an actual quantum computer?”
Ref: paper by Karth and Smith, In Proceedings of FDG’17

Group 4: “Unpaired Majorana fermions in quantum wires”
Ref: A Yu Kitaev “Unpaired Majorana fermions in quantum wires”, 2001 Phys.-Usp. 44 131

Group 5: Google’s paper on Quantum Supremacy?

Group 6: “Hybrid Quantum algorithm to classify Hermitian matrix definiteness”
Ref.: Gómez, Andrés, and Javier Mas. "Hybrid Quantum algorithm to classify Hermitian matrix definiteness." arXiv preprint arXiv:2009.04117 (2020).

Group 7: “Quantum Internet”
Ref: The quantum internet by H. J. Kimble, Nature 453, 1023-1030 (2010)

Week 13: The quantum 'Matrix': Quantum simulations and quantum sensing and metrology

Early ideas of quantum simulations

- Feynman in 1959: "Atoms on a small scale behave like nothing on a large scale, for they satisfy the **laws of quantum mechanics**. So, **as we go down and fiddle around with the atoms down there**, we are working with different laws, and **we can expect to do different things**."

- Feynman gave a lecture in 1981 on 'Simulating physics with computers"

"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

- Simulating quantum systems with classical computer requires exponential complexity ☹
- Proposed to use 'quantum simulators' instead ☺

$$|\Psi(0)\rangle \xrightarrow{\text{Time ordered } e^{-i\int^t dt' H(t')}} |\Psi(t)\rangle$$

Classical Church-Turing thesis [1936]

- Every 'function which would naturally be regarded as computable' can be computed by the universal Turing machine. (Universal Turing Machine can be used to simulate any other "classical computers".)

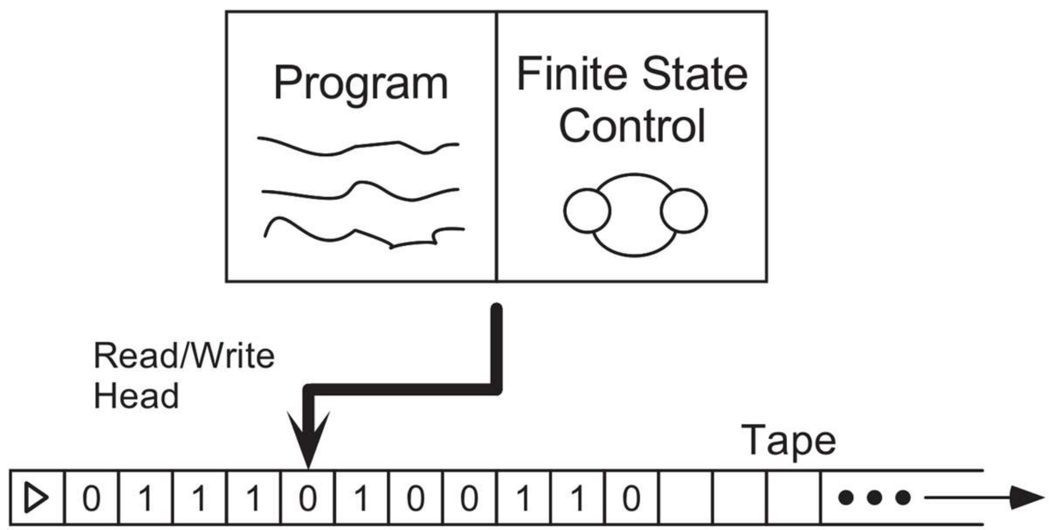


Fig. 3.1 of Nielsen & Chuang

Subsequent ideas of quantum simulations

- The Church-Turing-Deutsch Principle or quantum Church-Turing (Deutsch 1985)

Deutsch: could the laws of physics could be used to derive an even stronger version of the Church–Turing thesis?

  - Universal quantum computer (universal quantum Turing machine) is sufficient to efficiently simulate an arbitrary finite, realizable physical system

- Seth Lloyd [1996] showed Feynman's 1982 conjecture is correct that quantum computers can be programmed to simulate any local quantum system (containing few-particle interactions). Evolving in small time steps allows efficient simulation of time evolution; overall time needed grows only polynomially.

$$e^{iHt} = (e^{iH_1 t/n} \ldots e^{iH_\ell t/n})^n + \sum_{i>j} [H_i, H_j] t^2/2n + \sum_{k=3}^{\infty} E(k)$$

Lloyd's quantum simulations

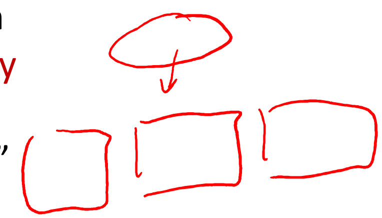
- Operations accessible consists of turning on and off Hamiltonians from a set:

$$\mathcal{S}_H \equiv \{\tilde{H}_1, \tilde{H}_2, \ldots, \tilde{H}_l\}$$

$A \rightarrow [\tilde{H}_i, \tilde{H}_j]$

Unitary operations are e^{iAt}, A in the algebra generated by $\mathcal{S}_H$

- He used the analogy of parking a car to describe how quantum simulator works. "By going forward and backing up a sufficiently small distance a large enough number of times, it is possible to parallel park in a space only ε longer than the length of the car."



$$e^{iHt} = (e^{iH_1 t/n} \ldots e^{iH_\ell t/n})^n + \underbrace{\sum_{i>j} [H_i, H_j] t^2/2n}_{\text{dominant error}} + \sum_{k=3}^{\infty} E(k)$$

Higher-order corrections

Some counting: classical vs. quantum

2^{N_q} comp

❑ Classical simulation

To get $\{\lambda_j\}$ $O(2^{3N_q})$ diff.

$H = \sum_j B_j \sigma_x^j$

$|\psi\rangle = \begin{pmatrix} |0\cdots0\rangle \\ |0\cdots1\rangle \\ \vdots \\ |1\cdots1\rangle \end{pmatrix}$

For N_q qubits, the state vector for system's wavefunction has 2^{N_q} components;

the evolution matrix is of size $2^{N_q} \times 2^{N_q}$ ➔ Exponential time complexity

$\Rightarrow H = \sum_j \lambda_j |n_j\rangle\langle n_j|$ $\rightarrow U(t) = \sum_j e^{-it\lambda_j} |n_j\rangle\langle n_j|$

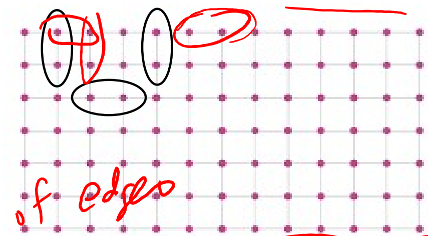
❑ Quantum simulation

$H = \sum_{j=1}^{l} H_j$

$$e^{iHt} = (e^{iH_1 t/n} \ldots e^{iH_\ell t/n})^n + \sum_{i>j} [H_i, H_j] t^2/2n + \sum_{k=3}^{\infty} E(k)$$

N_q^2 $\sim \varepsilon$ e.g. $\vec{S}_i \cdot \vec{S}_{i+1}$

- Assume each H_j acts on at most dimension m.
  Number of operations to simulate $e^{i H_j t/n}$ is m^2 at most. $m \times m$
- Each simulated n times qubits and there are l such terms → total number of operations is lnm^2.



- For desired error ε , error in each operation should be less than ε $/(lnm^2)$. l = # of edges e.g.
- For typical nearest neighbor or next-nearest neighbor interaction, $l \sim N_q$ (efficient)
- From the expansion, time steps $n \sim t^2/\varepsilon$ for time duration t . But duration for each operation is $t/n \sim 1/t$. ➔ total time complexity is $\sim t$ (linear in t but at most polynomial in N_q)

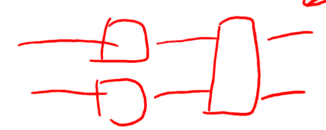
Digital quantum simulation

vs. analog (allows evolution $\boxed{e^{-iH_j\delta t}}$)

- One can also use a universal quantum computer to simulate evolution

construction of g. can be analog

operations from users' pt of view is digital



$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$

- Should further decompose the Trotter terms into unitary gates

decompose each into a sequence of gates

$$e^{iHt} = (e^{iH_1 t/n} \ldots e^{iH_\ell t/n})^n + \sum_{i>j} [H_i, H_j] t^2/2n + \sum_{k=3}^{\infty} E(k)$$

Cirac-Zoller criteria for quantum simulations

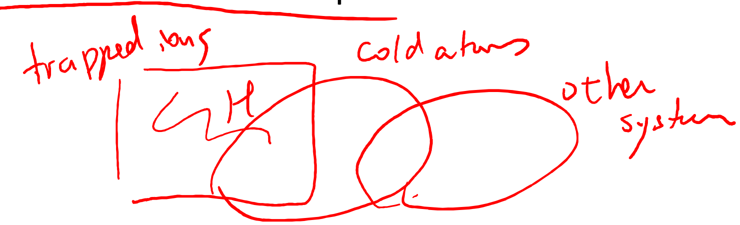
[Cirac & Zoller 2012]

1. Quantum system (bosons or fermions *or both* with or without spins)
➔ Contain a large number of degrees of freedom; particles can be confined in some region of space.

2. Initialization: Able to prepare (approximately) a known quantum state.

e.g. $H_j = \boxed{J_j}\, \vec{\sigma}_j \cdot \vec{\sigma}_{j+1} + \vec{B} \cdot \vec{\sigma}_j$

(3. Hamiltonian engineering
➔ Possible to engineer a set of interactions with external fields or between different particles, with adjustable values. They may involve a reservoir to simulate open-system dynamics. Among the accessible Hamiltonians there should be some that cannot be efficiently simulated (at present) with classical techniques.



Cirac-Zoller criteria (cont'd)

[Cirac & Zoller 2012]

4. Detection

➔ Able to perform measurements: individual (that is, addressing a few particular sites on the lattice) or collective. Ideally, one should be able to perform single-shot experiments that can be repeated several times; one would be able to determine not only $\langle S \rangle$, but also $\langle f(S) \rangle$

$H = \sum J_i \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}$

o o o · · · o o

e.g. ↑ ↑ ↑ ↑ ↑ ↓

→ ← ← → . .

5. Verification

➔ By definition no way of verifying the result if simulation is cannot be classically simulated efficiently. Should be a way of increasing the confidence in the result.

(for large system)

$\sigma_1 - \sigma_4 \Rightarrow \sigma_1 \sigma_2 \sigma_3 \sigma_4$

$\sigma_2 - \sigma_3$ (perturbation)

@ some special pts
⇒ classically computable

(a) Exactly solvable models in physics provide such a benchmark.
(b) Evolution may be run forwards and backwards in time to check if ends up in the initial state.
(c) Results of different methods and simulation systems could be compared.

Physical systems

[Cf. Prof. Schneble's Ultracold Quantum Systems Lab
Bloch, Dalibard & Nascimbène, Nat. Phys. 2012]

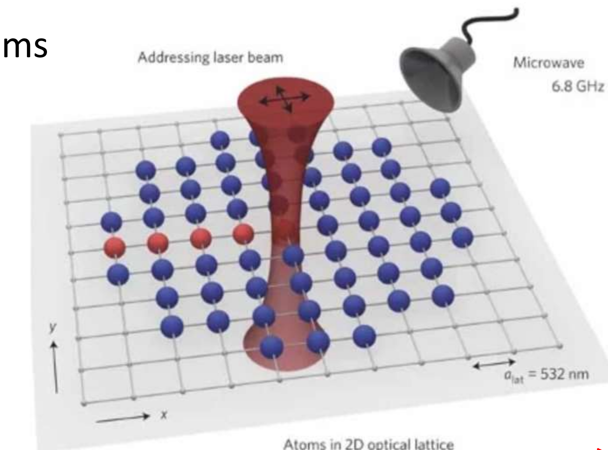
❑ **Ultracold quantum gases**

high degree of controllability, novel detection possibilities and extreme physical parameter regimes (compared to solid-state systems)

1. Feshbach resonances can be used to tune interaction between atoms
→ enables a simulator for other strongly interacting fluids

2. control of the energy landscape at the level of the single-particle Hamiltonian
→ trap atoms in optical lattices

3. Single-atom control and detection allow one to study the time evolution of these strongly correlated fluids



[Weitenberg, C. et al, Nature 2011]

4. Artificial gauge fields can be applied

$\left\{ \begin{array}{l} e^- \ \vec{E} \\ e\, \vec{v} \times \vec{B} \end{array} \right.$

Simulate gauge field (Electromagnetic wave)
neutral —— $\vec{B}, \vec{E}$ → Simulate charged particles

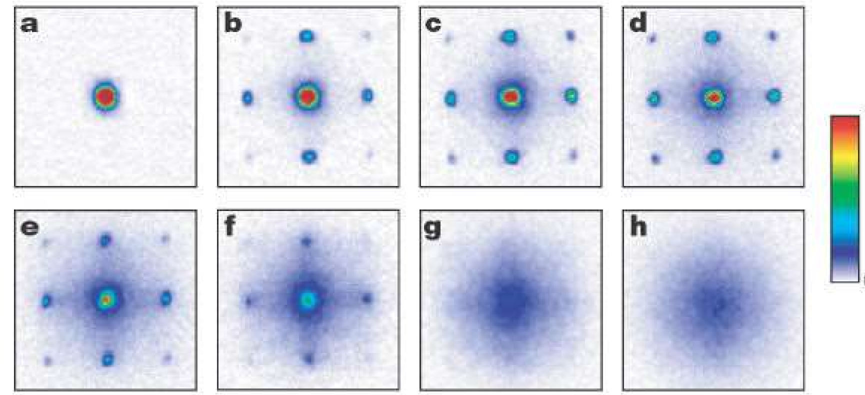
Article

Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

Markus Greiner, Olaf Mandel, Tilman Esslinger, Theodor W. Hänsch & Immanuel Bloch

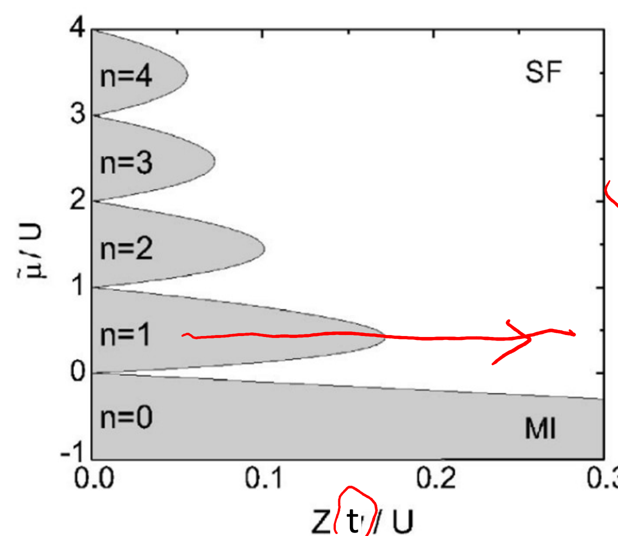
Nature **415**, 39–44 (03 January 2002)
doi:10.1038/415039a
Download Citation

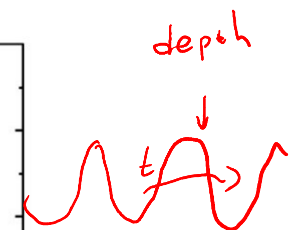
Received: 26 October 2001
Accepted: 29 November 2001
Published: 03 January 2002



Bose-Einstein condensate

$$H = -t\sum_{\langle i,j\rangle} \hat{b}_i^{\dagger}\hat{b}_j + \frac{U}{2}\sum_i \hat{n}_i\left(\hat{n}_i - 1\right) - \mu\sum_i \hat{n}_i .$$





Letter

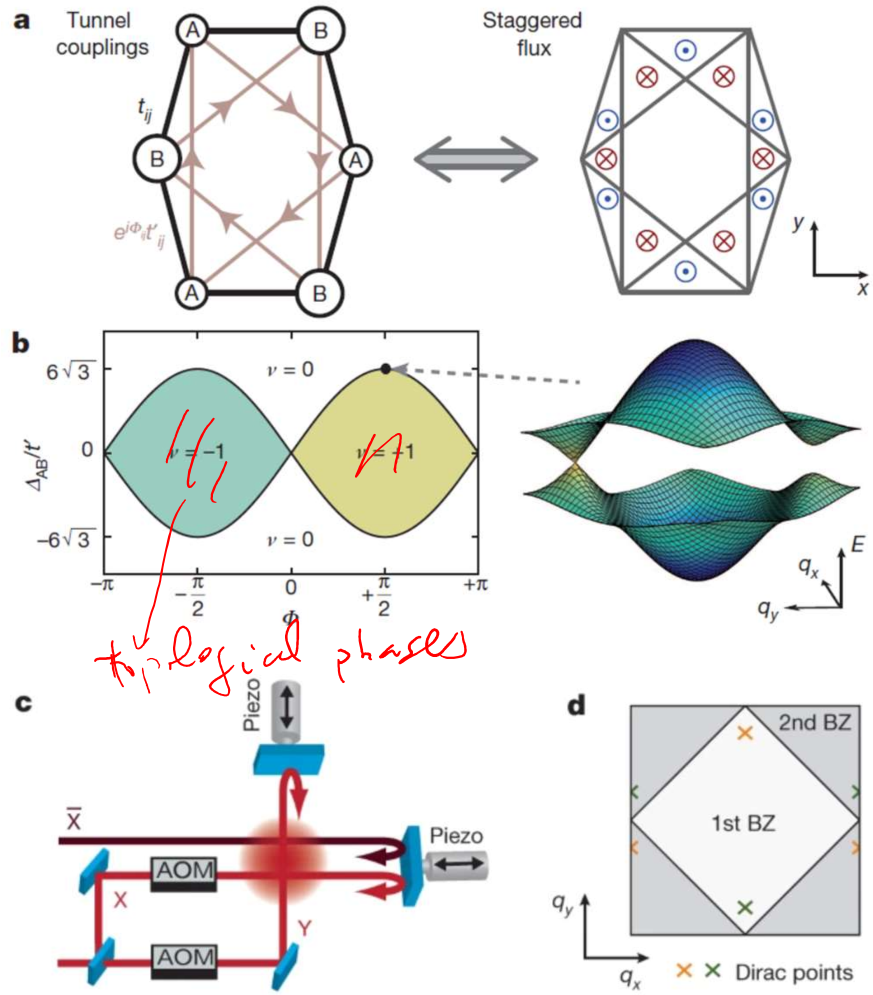
Experimental realization of the topological Haldane model with ultracold fermions

Gregor Jotzu, Michael Messer, Rémi Desbuquois, Martin Lebrat, Thomas Uehlinger, Daniel Greif & Tilman Esslinger

Nature **515**, 237–240 (13 November 2014)
doi:10.1038/nature13915
Download Citation
Quantum mechanics Ultracold gases

Received: 19 June 2014
Accepted: 29 September 2014
Published: 12 November 2014

$$\hat{H} = \sum_{\langle ij \rangle} t_{ij} \hat{c}_i^\dagger \hat{c}_j + \sum_{\langle\langle ij \rangle\rangle} e^{i\Phi_{ij}} t'_{ij} \hat{c}_i^\dagger \hat{c}_j + \Delta_{AB} \sum_{i \in A} \hat{c}_i^\dagger \hat{c}_i \qquad (1)$$



Article

Probing many-body dynamics on a 51-atom quantum simulator

Hannes Bernien, Sylvain Schwartz, Alexander Keesling, Harry Levine, Ahmed Omran, Hannes Pichler, Soonwon Choi, Alexander S. Zibrov, Manuel Endres, Markus Greiner ✉, Vladan Vuletić ✉ & Mikhail D. Lukin ✉

Nature **551**, 579–584 (30 November 2017)

doi:10.1038/nature24622

Download Citation

Quantum information | Quantum simulation

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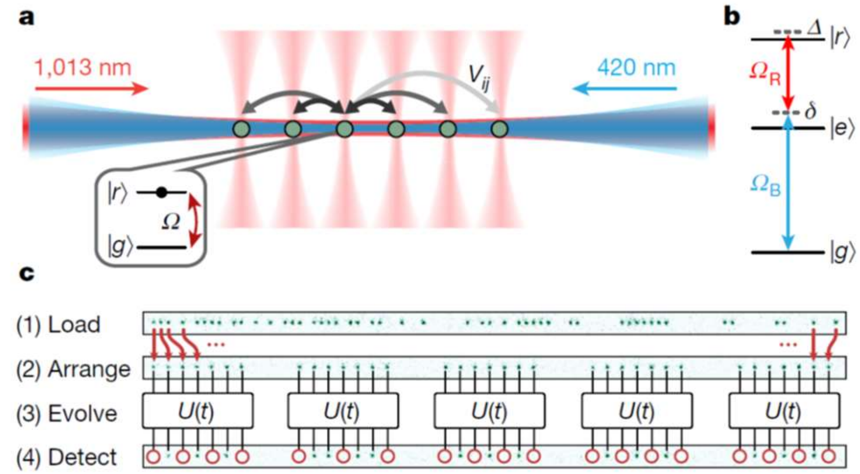
Published: 29 November 2017



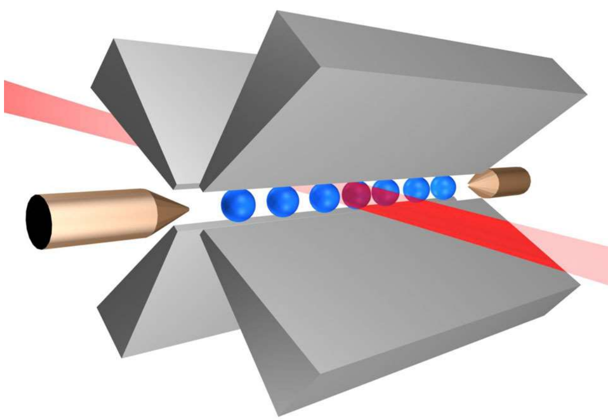
$$\frac{\mathcal{H}}{\hbar} = \sum_i \frac{\Omega_i}{2}\sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j$$

Physical systems

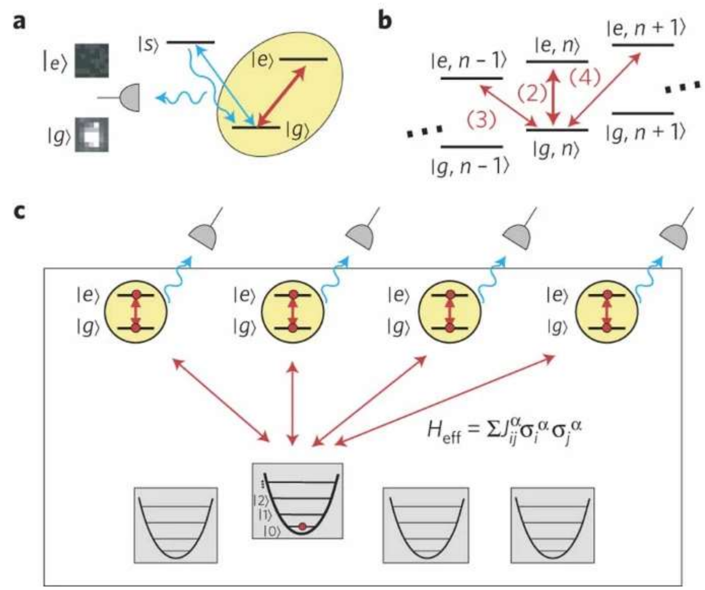
[Blatt and Roos, Nat. Phys. 2012]

- **Trapped ions**

Can be accurately controlled and manipulated; a large variety of interactions can be engineered with high precision; measurements of relevant observables can be obtained with nearly 100% efficiency



[Blatt's group, Univ. of Innsbruck]



14-Qubit Entanglement: Creation and Coherence

Thomas Monz, Philipp Schindler, Julio T. Barreiro, Michael Chwalla, Daniel Nigg, William A. Coish, Maximilian Harlander, Wolfgang Hänsel, Markus Hennrich, and Rainer Blatt

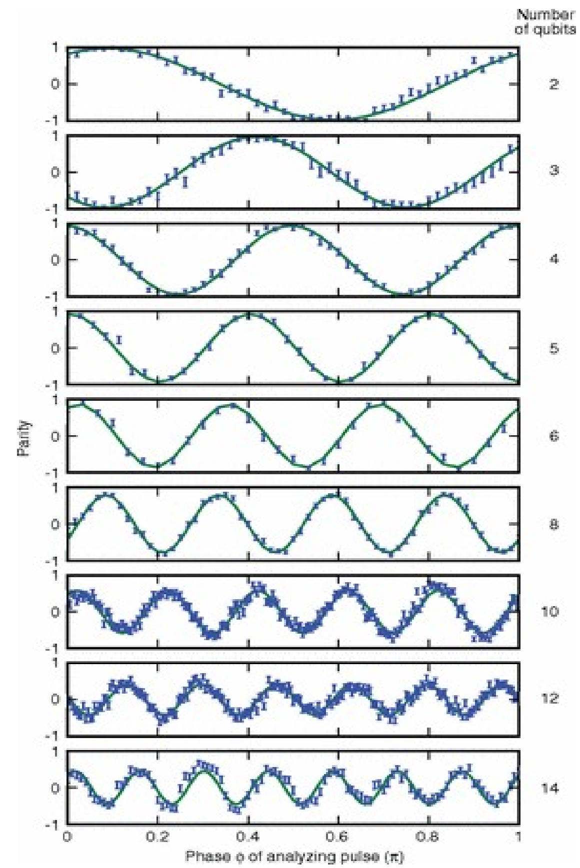
Phys. Rev. Lett. **106**, 130506 – Published 31 March 2011

ABSTRACT

3-qubit $|GHZ\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$

$(|0\cdots0\rangle + |1\cdots1\rangle)/\sqrt{2}$

We report the creation of Greenberger-Horne-Zeilinger states with up to 14 qubits. By investigating the coherence of up to 8 ions over time, we observe a decay proportional to the square of the number of qubits. The observed decay agrees with a theoretical model which assumes a system affected by correlated, Gaussian phase noise. This model holds for the majority of current experimental systems developed towards quantum computation and quantum metrology.



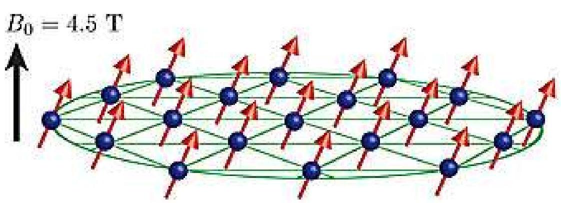
Parity oscillations observed on {2,3,4,5,6,8,10,12,14}-qubit GHZ states

Engineered two-dimensional Ising interactions in a trapped-ion quantum simulator with hundreds of spins

Joseph W. Britton, Brian C. Sawyer, Adam C. Keith, C.-C. Joseph Wang, James K. Freericks, Hermann Uys, Michael J. Biercuk & John J. Bollinger

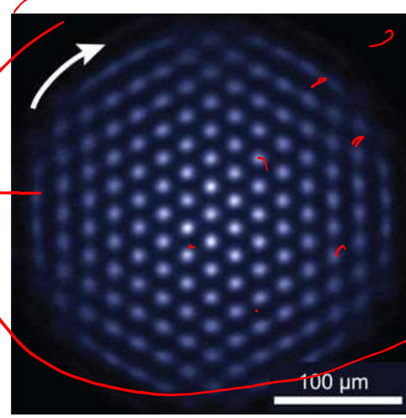
Nature **484**, 489–492(2012) | Cite this article

Here we demonstrate a variable-range Ising-type spin–spin interaction, $J_{i,j}$, on a naturally occurring, two-dimensional triangular crystal lattice of hundreds of spin-half particles (beryllium ions stored in a Penning trap)

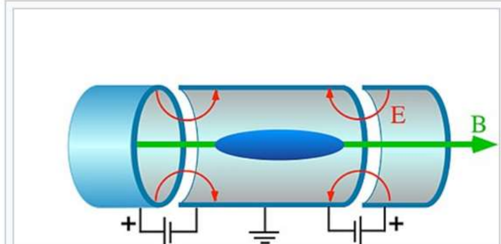


$$\begin{aligned} \hat{H}_B &= \textstyle\sum_i \vec{B}_\mu \cdot \hat{\vec{\sigma}}_i \\ \hat{H}_I &= \tfrac{1}{N} \textstyle\sum_{i<j} J_{i,j} \hat{\sigma}_i^z \hat{\sigma}_j^z . \end{aligned}$$

~300 ions



enlarge



A cylindrical version of Penning trap, with open ends to permit through flow

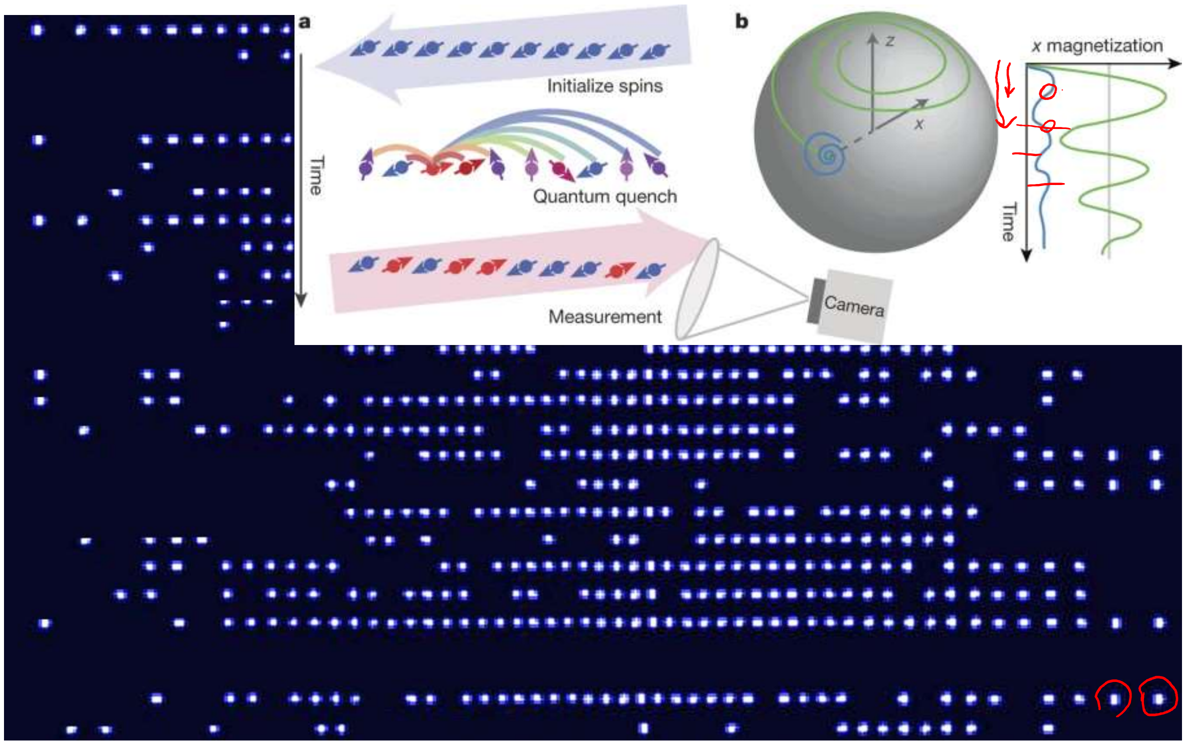
[Penning trap; see Wikipedia]

LETTER

doi:10.1038/nature24654

Observation of a many-body dynamical phase transition with a 53-qubit quantum simulator

J. Zhang[1], G. Pagano[1], P. W. Hess[1], A. Kyprianidis[1], P. Becker[1], H. Kaplan[1], A. V. Gorshkov[1], Z.-X. Gong[1]† & C. Monroe[1,2]



"Use a quantum simulator composed of up to 53 qubits to study non-equilibrium dynamics in the transverse-field Ising model with long-range interactions.

➔ Observe a dynamical phase transition after a sudden change of the Hamiltonian"

$$H = \sum_{i<j} J_{ij}\sigma_i^x\sigma_j^x + B_z\sum_i \sigma_i^z \qquad (1)$$

analog evolution +

single-shot measurement

$\sum \langle S_z^i \rangle$ / $\sum \langle (S_z^i)^n \rangle$,

Observation of a discrete time crystal

J. Zhang[1], P. W. Hess[1], A. Kyprianidis[1], P. Becker[1], A. Lee[1], J. Smith[1], G. Pagano[1], I.-D. Potirniche[2], A. C. Potter[3], A. Vishwanath[2,4], N. Y. Yao[2] & C. Monroe[1,5]

Floquet (periodic drive)

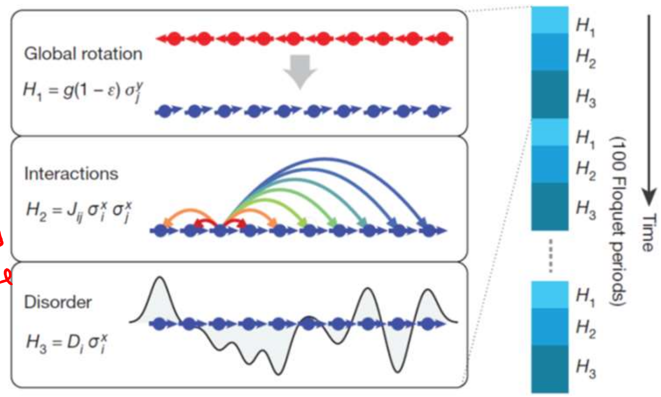
doi:10.1038/nature21413

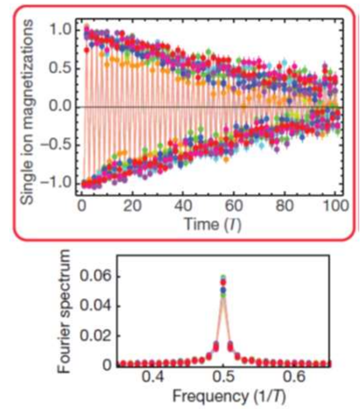
[On trapped ions]

$$H=\begin{cases} H_1=g(1-\varepsilon)\sum_i \sigma_i^y & \text{time } t_1 \\ H_2=\sum_i J_{ij}\sigma_i^x\sigma_j^x & \text{time } t_2 \\ H_3=\sum_i D_i\sigma_i^x & \text{time } t_3 \end{cases}$$

one cycle

10 Yb+ ions





Observation of discrete time-crystalline order in a disordered dipolar many-body system

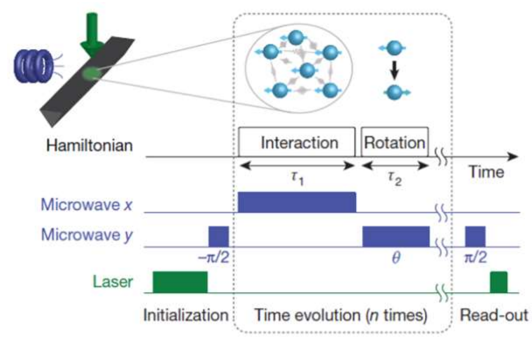
Soonwon Choi[1*], Joonhee Choi[1,2*], Renate Landig[1*], Georg Kucsko[1], Hengyun Zhou[1], Junichi Isoya[3], Fedor Jelezko[4], Shinobu Onoda[5], Hitoshi Sumiya[6], Vedika Khemani[1], Curt von Keyserlingk[7], Norman Y. Yao[8], Eugene Demler[1] & Mikhail D. Lukin[1]

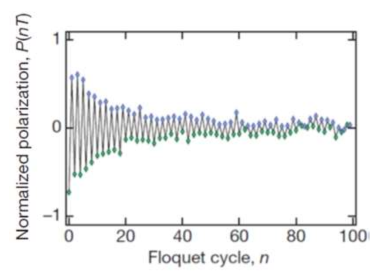
doi:10.1038/nature21426

[On NV centers in diamonds]

$$H(t)=\sum_i \Omega_x(t)S_i^x+\Omega_y(t)S_i^y+\Delta_i S_i^z + \sum_{ij}\left(J_{ij}/r_{ij}^3\right)\left(S_i^xS_j^x+S_i^yS_j^y-S_i^zS_j^z\right)$$

[system: high centration NV centers (45ppm)]





Other physical systems

- ❑ Already saw NV centers in diamond
- ❑ Superconducting qubits have been deployed in quantum computers (e.g. IBM, Google, Rigetti, etc.)

  ➔ Already saw their potential and current limitation as a universal quantum computer (and a quantum annealer)

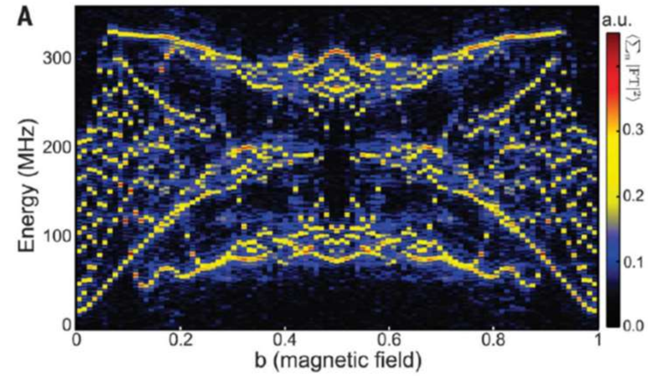
- ❑ Photonic systems

REPORT

Spectroscopic signatures of localization with interacting photons in superconducting qubits

P. Roushan[1,*,†], C. Neill[2,†], J. Tangpanitanon[3,†], V. M. Bastidas[3,†], A. Megrant[1], R. Barend...
+ See all authors and affiliations

Science 01 Dec 2017:
Vol. 358, Issue 6367, pp. 1175-1179
DOI: 10.1126/science.aao1401



Hofstadter butterfly; with 9 sc qubits

Using a chain of nine superconducting qubits, we implement a technique for resolving the energy levels of interacting photons. We benchmark this method by capturing the main features of the intricate energy spectrum predicted for two-dimensional electrons in a magnetic field—the Hofstadter butterfly.

Each qubit can be thought of as a nonlinear photonic resonator in the microwave regime with the Hamiltonian:

$$H_{\mathrm{BH}} = \sum_{n=1}^{9} \mu_n a_n^\dagger a_n + \frac{U}{2}\sum_{n=1}^{9} a_n^\dagger a_n (a_n^\dagger a_n - 1) + J \sum_{n=1}^{8} a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \qquad (2)$$

$$H_{\mathrm{Harper}} = \Delta \sum_{n=1}^{9} \cos(2\pi n b) a_n^\dagger a_n + J \sum_{n=1}^{8} a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1}$$

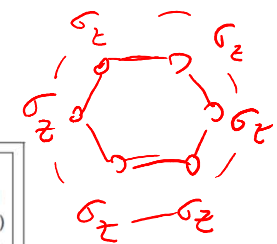
nature chemistry
ARTICLES
PUBLISHED ONLINE: 10 JANUARY 2010 | DOI: 10.1038/NCHEM.483

– photonic system

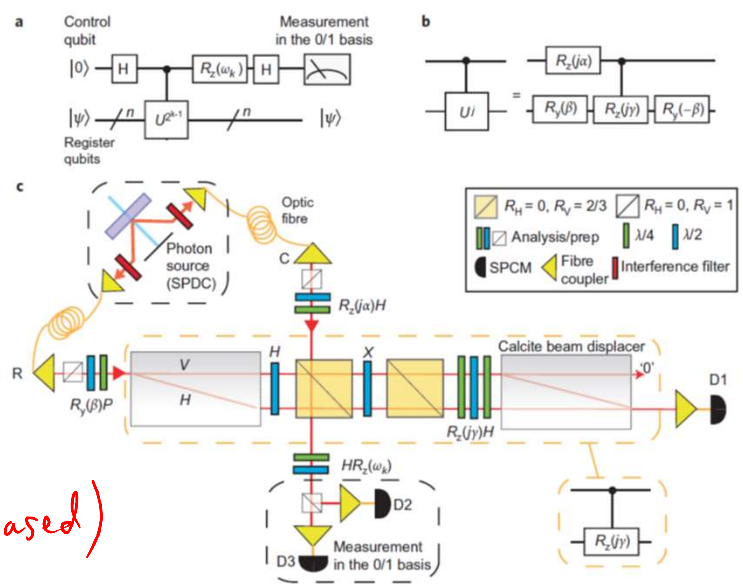
study Boson Sampling Problems

Towards quantum chemistry on a quantum computer

B. P. Lanyon[1,2]*, J. D. Whitfield[4], G. G. Gillett[1,2], M. E. Goggin[1,5], M. P. Almeida[1,2], I. Kassal[4], J. D. Biamonte[4]†, M. Mohseni[4]†, B. J. Powell[1,3], M. Barbieri[1,2]†, A. Aspuru-Guzik[4]* and A. G. White[1,2]



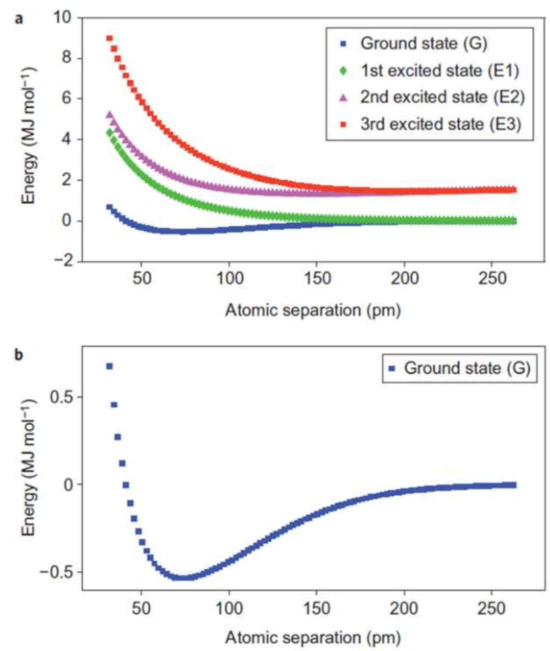
Photonic system:



digital

(≈ gate-based)

for hydrogen molecule



analog

~~SH~~

e.g. $\langle S_z \rangle$

e.g. expand cloud