Unit 3: Information is Physical

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In this unit, we discuss the physics behind various realizations of quantum information carriers. We will learn the physics language and some related mathematics.

Learning outcomes: You'll be able to know various physical systems and candidates to realize qubits and quantum computers.

Note that there are many pictures that were used from other sources on the web or papers. Hopefully, these will be acknowledged appropriately as current notes were put together in a haste.

I. INTRODUCTION

We need physical systems to encode, carry and process quantum information, so we will learn a few different systems and how to use them to realize qubits. These include superconducting qubits, solid-state spin qubits, photons, trapped ions, and topological qubits (p-wave superconductors, fractional quantum Hall systems, topological insulators, etc.).

What properties do these need to have in order to be a good platform for quantum information processing or even for building a full-fledge quantum computer? In 2000, David DiVincenzo (then at IBM) came up a list, which is now referred to as the DiVincenzo's criteria [1] (see also an article in Nature Reviews Physics [2]) .

- A scalable physical system with well-characterized qubits.
- The ability to initialize the state of the qubits to a simple fiducial state, such as $00\ldots0$.
- Long relevant decoherence times (relaxation T_1, dephasing T_2), much longer than the gate operation time.
- A "universal" set of quantum gates: e.g. Hadamard gate, T gate and CNOT gates (discussed more in later lectures).
- A qubit-specific measurement capability.

Two more items that are needed for quantum communication.

- The ability to interconvert stationary and flying qubits.
- The ability to faithfully transmit flying qubits between specified locations.

There have been many commerical quantum computers emerging (some with qubit number over one hundred) and they are available for users to try and test the performance [3]. The world of quantum computing is now at a much advanced place than, say, 20 years ago, when I was still a graduate student and in the days of one and two qubits.

II. VARIOUS PHYSICAL SYSTEMS

A. Light & photons [4, 5]. Advantage: clean, low decoherence; Disadvantage: hard to entangle, loss of photons can be an issue. See Fig. 1 for an overview of different photonic qubit realizations.

Even for light/photons, there are several degrees of freedoms that one can use.

1. Polarization (direction of Electric field): Horizonal 0, Vertical 1 (Z-basis); Diagonal , Antidiagonal (X-basis); Right circular, Left circular (Y-basis). Their qubit sphere is also called Poincaré sphere; see, e.g., `https://en.wikipedia.org/wiki/Polarization_(waves)#Poincar_sphere`.

2. Time bins (pulse position): use a Mach-Zehnder interferometer to split the pulse.

3. Dual-rail encoding: Used in Knill-Laflamme-Milburn scheme of linear-optic quantum computation [6]. Essentially, there are two 'modes' that a photon can reside in (note similar to double slits). We can define (1) $|0\rangle$ as the case where there is only one photon in the first mode and zero photon in the second mode, and (2) $|1\rangle$ as the case where there is only one photon in the second mode and zero photon in the first mode.

4. Continuous-variable: e.g. coherent state. The so-called 'cat' states or codes are a popular subject for quantum computation in superconducting qubits coupled via cavity modes [7]; see also a news article at Yale University's website: `https://news.yale.edu/2020/08/12/yale-quantum-researchers-create-error-correcting-cat`.

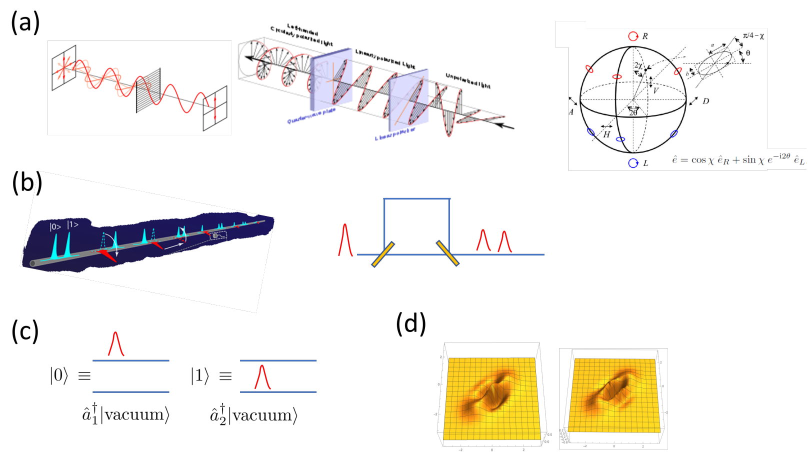


FIG. 1. Various qubit implementations with light. (a) Polarization, (b) Time bins, (c) Dual rails, and (d) Coherent state or continuous variables.

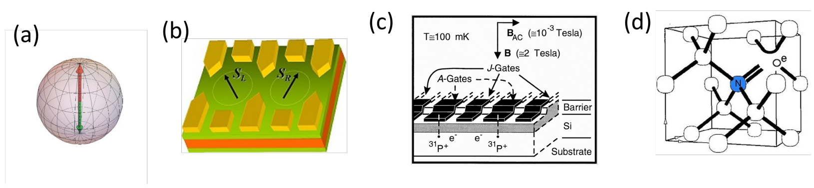


FIG. 2. Various qubit implementations with spins. (a) Bloch sphere of a spin, (b) Quantum dots, (c) Phosperous donors on silicon, and (d) Nitrogen-vacancy centers in diamond.

Photons are bosons and there are operators to describe their creation and annihilation; see below in Eq. (1).

B. Spins. The examples are electron spins, diamond NV center [8], quantum dots [9], nuclear spins [10], etc. In general, spin angular momentum operators are associated with generators of rotation (setting $\hbar \equiv 1$ for convenience) :

$$(-i)S_\alpha = \frac{d}{d\theta}\Big|_{\theta\to 0} R_\alpha(\theta),\ [S_x, S_y] = iS_z,\ [S_y, S_z] = iS_x, \text{ etc.}$$

We have seen Pauli matrices and they are related to spin-1/2 particles that two states up ($S_z = +1/2$) and down ($Sz = -1/2$), and the spin operators are related to Pauli matrices in this way:

$$\vec{S}_\alpha = \frac{1}{2}\vec{\sigma}_\alpha = \frac{1}{2}(X, Y, Z).$$

Advantage: spin-1/2 is precise 2-level system; controllable by magnetic field. Disadvantage: solid-state environment is noisy and the spins have short coherence time. See Fig. 2 for an overview of various systems with spins.

Researchers have grown quantum dots using semiconductors by electrically confined an area that form the shape of a dot and such a dot can host an effective electron [9]. The two electrons on neighboring dots interact via Heisenberg coupling, whose strength can be via electronic back gates: $\hat{H} = J\vec{S}_L \cdot \vec{S}_R$. Phosphorus donors on pure silicon (nuclear spin) was also proposed for implementing qubits [10], which is a system focused by many researchers in Australia. Both systems have the advantage of integrating with current silicon technology.

Nitrogen-vacancy (NV) center in diamond [8, 11]: e.g. the negatively charged state N-V$^-$ electron spins (ground state with $S = 1$) can be manipulated by electric field and magnetic field. Coherence time is micro- to milliseconds and can be made longer by "dynamical decoupling"). It is also proposed to also use a NV center and nearby ^{13}C nuclear spin for quantum operation.

C. Trapped ions. The early design to trap ions is to use the so-called Paul trap, which consists of four rods at certain DC and AC potentials, as shown in Fig. 3, which are

$$\Phi_{\rm dc}(x, y, z) = \kappa U_0[z^2 - (x^2 + y^2)]/2,\ \Phi_{\rm rf}(x, y) = (V_0 \cos\Omega_T t + U_r)(1 + (x^2 - y^2)/R^2)/2.$$

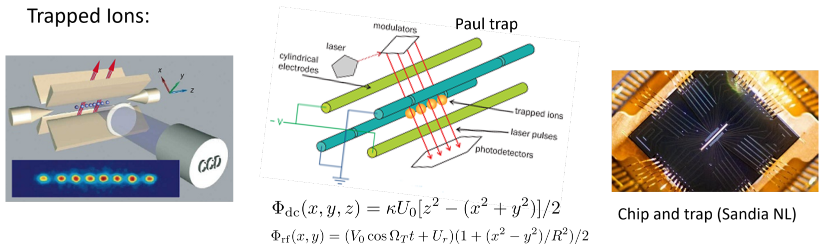


FIG. 3. Trapped ions in a Paul trap.

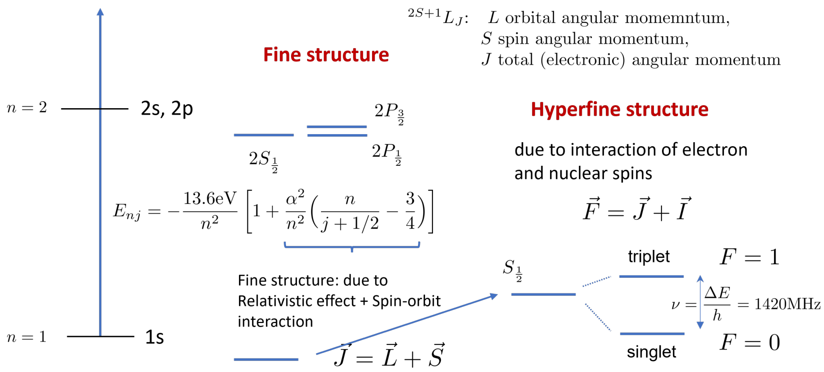


FIG. 4. Illustration of eletronic levels, fine and hyperfine structure of a hydrogen atom (not to scale).

The ions used usually have one electron at the outermost shell, such as $^9\text{Be}^+$ (Wineland's goup), $^{40}\text{Ca}^+$ (Blatt's group); $^{133}\text{Ba}^+$, and $^{171}\text{Yb}^+$ (used by IonQ, Monroe's group), etc.

To understand the physical states (i.e. hyperfine states) used, we need to understand state in the electronic levels, fine and hyperfine structures. We will illustrate these with the hydrogen. Most of you know energy levels of a hydrogen atom and some orbitals, $1s, 2s, 2p, 3s, 3p, 3d$, etc. (where $n = 1, 2, 3..$ is the so-called principal quantum number). See Fig. 4.

In physics or chemistry, the energy levels are usually denoted by a symbol $^{2S+1}L_J$, where L is the orbital angular momemntum, S the spin angular momentum, and J total (electronic) angular momentum

Fine structure refers to finer details of the electronic levels that are split by effects such as relativistic effect and spin-orbit interaction. In this case, the total angular momentum is conserved: $\vec{J} = \vec{L} + \vec{S}$.

$$E_{nj} = -\frac{13.6\text{eV}}{n^2}\Big[1 + \underbrace{\frac{\alpha^2}{n^2}\Big(\frac{n}{j+1/2} - \frac{3}{4}\Big)}_{\text{Relativistic effect \&Spin-orbit interaction}}\Big]$$

Hyperfine structure: further splitting of levels is due to interaction of electrons with nuclear spins. $\vec{F} = \vec{J} + \vec{I}$, where $\vec{I}$ is the nuclear spin operator. For further discussions on building up qubits and gates in trapped ions, see Ref. [12].

D. Trapped neural atoms. See Fig. 5.

1. Optical Lattice: various alkali atoms are used. Two states in the ground hyperfine levels can be used to encode a qubit.

2. Rydberg atoms (electronic states with high principal n number). For hydrogen-like atoms, radius is roughly

$$r_n \sim n^2 a_{\rm Bohr}/Z.$$

Two hyperfine ground states $|0\rangle$ and $|1\rangle$ and use a Rydberg state $|r\rangle$ to construct a controlled gate via the Rydberg blockade.

For further discussions on neutral atoms for quantum computing, see, e.g., Refs. [13, 14].

E. Superconducting qubits. Superconducting qubits: (a) Phase; (b) Flux; (c) Charge; (d) Transmon/Xmon. (See Figs. 6 & 7). Crucial ingredient: Josephson junction that gives rise to nonlinear inductance; see Fig. 6d.

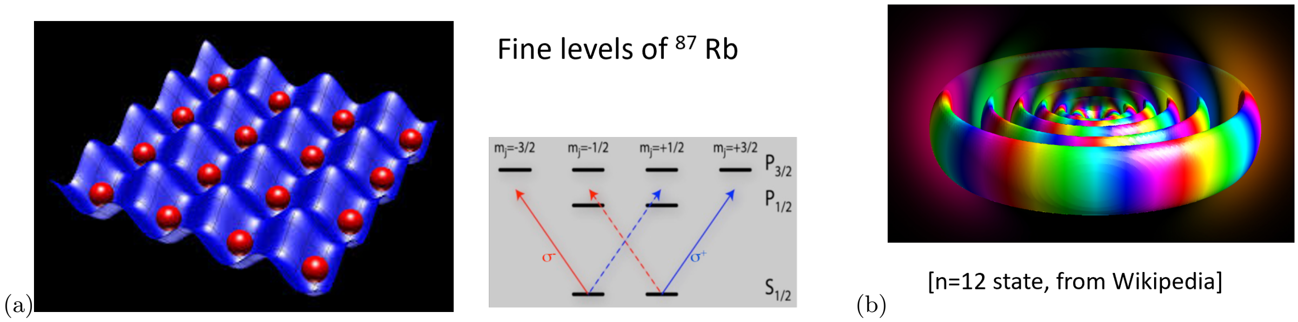


FIG. 5. Illustration of neural atoms: (a) Alkali atoms (e.g. Rb) in optical lattice and (b) Rydberg atoms.

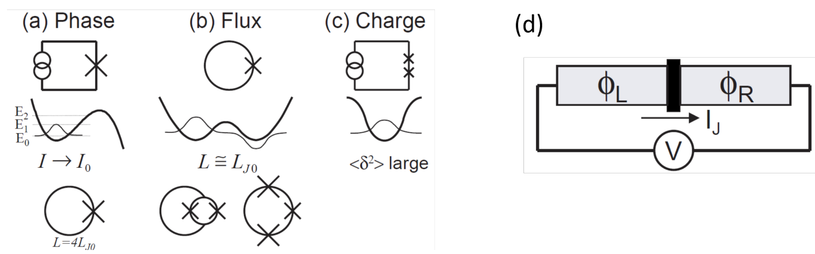


FIG. 6. Illustration of several designs of superconducting qubits (a-c), and illustration of a Josephson junction (d).

Here is the Josephson relations:

$$I_J = I_0 \sin(\phi_L - \phi_R) = I_0 \sin\delta, \;\; V = \frac{\Phi_0}{2\pi}\frac{d\delta}{dt}, \quad \Phi_0 \equiv \frac{h}{2e},$$

where δ is the phase difference between two superconductors across the junction. From the relation between voltage, inductance and current rate, $V = L\frac{dI}{dt}$, we get for the Josephson junction,

$$V = L_J \frac{dI_J}{dt} = L_J I_0 \cos\delta \cdot V\frac{2\pi}{\Phi_0}, \;\; L_J = \frac{\Phi_0}{2\pi I_0 \cos\delta} = \pm\frac{\Phi_0}{2\pi\sqrt{I_0^2 - I_J^2}}.$$

Energy stored in junction:

$$U = \int V I_J \, dt = -\frac{\Phi_0 I_0}{2\pi}\cos\delta = -E_J \cos\delta,$$

where $E_J = \frac{\Phi_0 I_0}{2\pi}$ is the Josephson energy.

The proposal of the transmon originally came from the work of Koch and collaborators [15].As there are metalic plates and hence capacitors, another contribution to the total energy is the so-called 'charging' energy: $\frac{Q^2}{2C}$. In superconductors, two electrons combine to form a Cooper pair with charge $2e$, and thus this energy is also written as $4e^2\hat{n}^2/(2C) = E_c\hat{n}^2$, where $\hat{n}$ denote the number operator for the Cooper pair. We often add a charge offset to the energy: $\hat{n}^2 \to (\hat{n} - n_g)^2$, so the total Hamiltonian is

$$\hat{H}(n_g) = 4E_c(\hat{n} - n_g)^2 - E_J \cos\hat{\delta}.$$

In quantum mechanics, the position operator $\hat{x}$ and momentum operator $\hat{p}$ do not commute: $[\hat{x}, \hat{p}] = i\hbar$. This means $\hat{o} = (\hbar/i)d/dx$. Here, for the two operators $\hat{n}$ and $\hat{\delta}$, they satisfy $[\hat{\delta}, \hat{n}] = i$. What this means is that the Hamiltonian acts on a state described by a wavefunction in δ is

$$\hat{H}(n_g) = 4E_c(-id/d\delta - n_g)^2 - E_J \cos\hat{\delta}.$$

But we will not explicitly solve the equation $\hat{H}\psi(\delta) = E\psi(\delta)$. Solving this will give discrete energy levels for the system; see Fig. 8 for some examples. For further discussions on superconducting quantum computing systems, see, e.g., Refs. [16–18] and Les Houches Lecture Notes "Superconducting Qubits and the Physics of Josephson Junctions," by J. Martinis and K. Osborne at `https://web.physics.ucsb.edu/~martinisgroup/classnotes/finland/LesHouchesJunctionPhysics.pdf`.

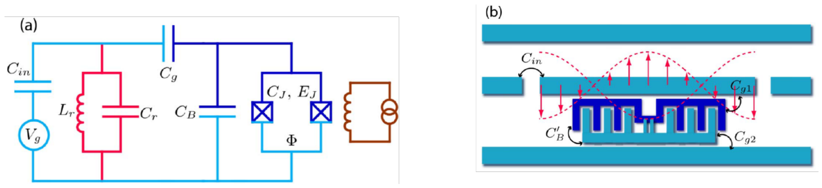


FIG. 7. Illustration of the design of a transmons. Koch et al.

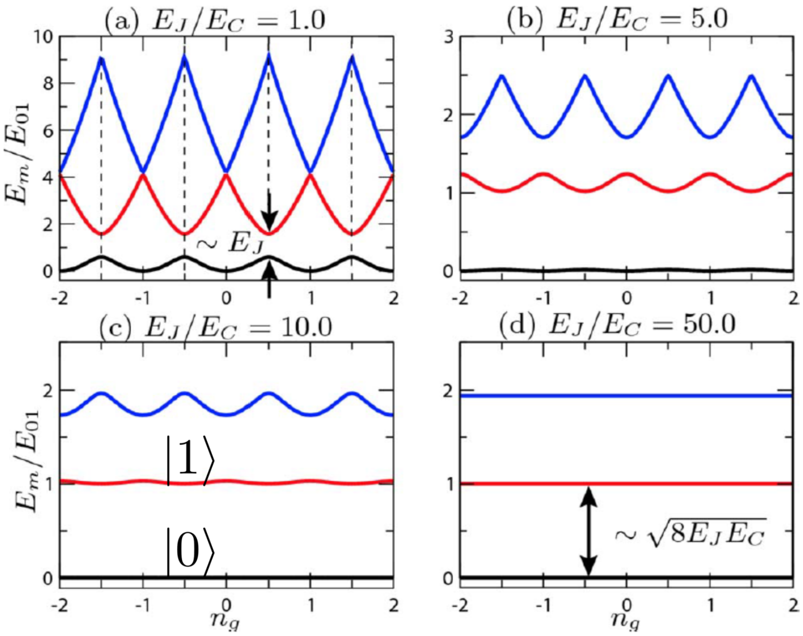


FIG. 8. Illustration of the energy levels of a transmons; pictures from Koch et al. [15].

III. TOPOLOGICAL QUBITS

To understand how topological qubits utilize anyons, we will step back and understand the properties of bosons and fermions.

Bosons, fermions and anyons. Fermions, such as electrons, cannot occupy the same state and their wavefunction gives a minus sign under particle exchange,

$$\Psi_F(x_1, \ldots, x_j, \ldots, x_k, \ldots) = -\Psi_F(x_1, \ldots, x_k, \ldots, x_j, \ldots).$$

Such antisymmetry is encoded in the anticommulation relations of fermion annihilation and creation operators,

$$\{\hat{c}_i, \hat{c}_j\} = \{\hat{c}_i^\dagger, \hat{c}_j^\dagger\} = 0, \ \{\hat{c}_i, \hat{c}_j^\dagger\} = \delta_{ij},$$

where indices i and j denote the location or other degrees of freedom such as momentum.

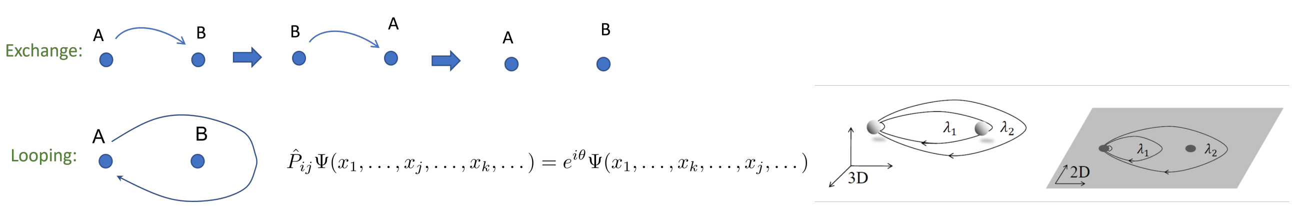


FIG. 9. (a) Illustration of exchange of two particles. (b) Comparision of two and three dimensions. [Need to credit the source of the picture.]

Bosons, such as photons, prefer to occupy the same state and their wavefunction is the same under particle exchange,

$$\Psi_B(x_1,\ldots,x_j,\ldots,x_k,\ldots) = \Psi_B(x_1,\ldots,x_k,\ldots,x_j,\ldots).$$

Such exchange symmetry is encoded in the commutation relations,

$$[\hat{b}_i,\hat{b}_j] = [\hat{b}_i^\dagger,\hat{b}_j^\dagger] = 0,\ [\hat{b}_i,\hat{b}_j^\dagger] = \delta_{ij}. \quad (1)$$

As we can see from Fig. 9, two exchanges between two particles equal a particle loops around the other (up to a translation). Thus, $\hat{P}_{ij}^2\Psi = \Psi$. In three dimensions, the loop can be continuously deformed to a point, and therefore $\hat{P}_{ij}^2 = I$. There are two types of eigenstates: $e^{i\theta} = \pm 1$ and they correspond to bosons and fermions, respectively.

However, in two dimensions, the loop cannot shrink to a point without crossing the other particle. Thus there is no constraint on particle statistics. Any phase θ is allowed, and the associated particles are called anyons.

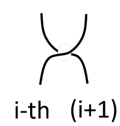
Anyons do appear in condensed-matter systems, at least, theoretically. Intrinsic topological phases harbor anyons and their braiding (braid group) gives rise to quantum gates [19]. We will discuss more on these in later lectures. Here, we list a few systems that possess anyons as excitation.

- Fractional Quantum Hall system.
- Quantum spin liquids. One possible example is the Heisenberg spin model on the kagome lattice. Another one is the Kitaev's toric code model.
- 2D $p + ip$ superconductor, which has Majorana fermions. They appear at the vortex cores and are denoted by γ's [20].

The effect of exchanging or braiding two vorices gives the braiding of two Majoranas and for two such particles, the mathematical effect is

$$\gamma_1 \to \gamma_2,\ \gamma_2 \to -\gamma_1.$$

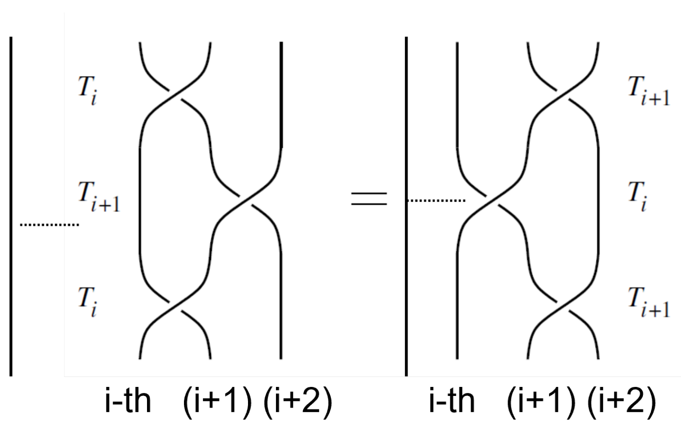
Braid groups. We use T_i to represent braiding of i-th and (i+1)-th threads:



They form a group but there is some constraint:

$$T_i T_{i+1} T_i = T_{i+1} T_i T_{i+1},$$

as illustrated in the graphical equation (or braid equation) below.



The goal is to find anyons whose braid group gives rise to 'universal' gates so that any quantum circuit can be expressed in terms of elements in the braid group. Whether this can be done depends on the type of anyon models. For example, Fibonacci anyons are universal, but Ising anyons are not.

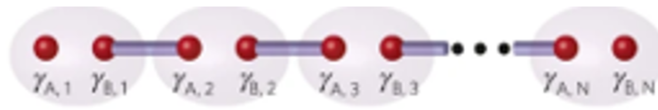


FIG. 10. Illustration of the Kitaev's chain in the topological phase, i.e. with Majorana zero modes at the ends. Picture taken from Ref. [21].

More on Majorana fermions/zero modes. In condensed matter systems, Majorana fermions are more approriately called Majorana zero modes. There are "Majorana" operators that we can define from fermion operators,

$$\hat{\gamma}_B \equiv \hat{c} + \hat{c}^\dagger, \ \hat{\gamma}_A \equiv (\hat{c} - \hat{c}^\dagger)/(i).$$

These Majorana operators satisfy

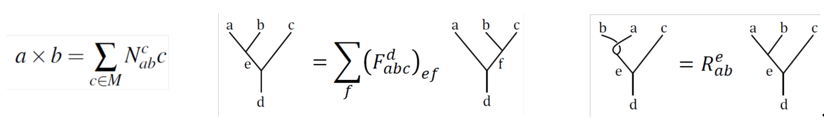
$$\hat{\gamma}_A^2 = I = \hat{\gamma}_B^2, \ \hat{\gamma}_B^\dagger = \hat{\gamma}_B, \ \hat{\gamma}_A^\dagger = \hat{\gamma}_A.$$

They are like 'halves' of a fermion. One example that Majorana fermions can arise is the Kitaev's fermion chain (with p-wave pairing), and the simplest model is

$$H = -\sum_{x=1}^{N-1} \hat{c}_x^\dagger \hat{c}_{x+1} + \hat{c}_x \hat{c}_{x+1} + h.c. = -i \sum_{x=1}^{N-1} \hat{\gamma}_{B,x} \hat{\gamma}_{A,x+1}.$$

Despite that this is a one-dimensional structure, it has been proposed to use T-junctions to braid Majorana anyons [21]. For further discussions on Majorana anyons and quantum computation, see, e.g. Ref. [22]. We will discuss topological quantum computation in later lectures; but you can read an article on this website: `https://ncatlab.org/nlab/show/topological%20quantum%20computation`.

Why topological qubits? Topological quantum computation is robust against noise (does not need active error corrections) [more on this in later lectures]. There are beautiful and elegant mathematics involved, e.g.



Qubits are encoded in different 'fusion channels' (i.e. analogous to different paths to arrive at a point). Quantum gates are implemented via braiding anyons. The above mathematics plus suitable choice of fusion channels give rise to quantum gates on qubits. To read out results, we need to bring anyons together and try to annihilate them, e.g. in pairs, to measure what is left. Despite the beauty in the formalism, however, it is still very challenging to realize good topological qubits [there is effort from academia and industry such as Microsoft (both theory and experiments)] and so far there is no real demonstration of topological qubits in experiments.

IV. CURRENT QUANTUM COMPUTERS

Some existing quantum computers. In Fig. 11, we display pictures of quantum computers made by various companies. Currently, there are many more and the number of qubits keeps increasing. There are many companies focusing different physical qubits and mechanisms of coupling. However, these are still noisy and are referred to as noisy intermediate-scale quantum (NISQ) devices [23].

Several implementations of quantum computers are done with solid-state devices, and it is a nontrivial task to engineer the interface between the quantum part of the system and the classical part of the control; see e.g. Ref. [24].

One of the most impressive experimental works done is the Google Quantum AI's demonstration of 'quantum supremacy' on random quantum circuits [25]. But there are other important milestone works as well, such as Boson Sampling.

Physical systems for quantum simulations. Physical systems can also be used to simulate Hamiltonians and their dynamics; this is an idea proposed by Feynman in the 1980s [26]. There were various experiments demonstrating this. For example, cold atoms trapped in optical lattices have been used to simulate a lot of physics, such as Mott-Superfluid transition, Hubbard models, Haldane's honeycomb models. These can be regarded as analogue quantum simulators.

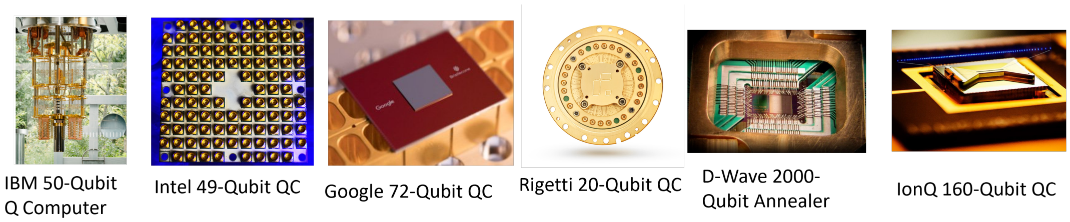


FIG. 11. Illustration of various quantum computers.

V. CONCLUDING REMARKS

In this unit, we have discussed various physical systems, but only superficially. Each system will require a lecture or two on its own.

It is a good time to check whether you have achieved the following Learning Outcomes:
After this Unit, you'll be able to know various physical systems and candidates to realize qubits and quantum computers.

Suggested reading: N&C chap 7 and various papers listed on the course website: `http://insti.physics.sunysb.edu/~twei/Courses/Fall2021/PHY682/#Unit03`.

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