Unit 0: Appetizer of quantum phenomena

Tzu-Chieh Wei
C. N. Yang Institute for Theoretical Physics and Department of Physics and Astronomy, State University of New York at Stony Brook, Stony Brook, NY 11794-3840, USA
(Dated: August 24, 2025)

In this unit, we will give a very brief taste of a couple of quantum phenomena. These serve as an appetizer of this quantum information science course. You may feel that quantum physics is weird and mysterious. This was certainly the case to researchers in the early 1900s and later. As we now enter a new era where quantum information processing becomes more and more realistic, we will turn the mysterious parts of quantum mechanics into something useful, which we will study in later units.

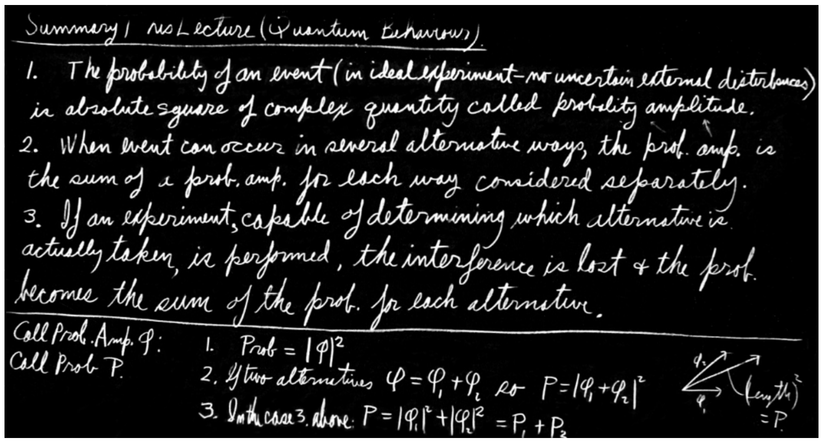

FIG. 1. Illustration of some quantum mechanical rule: a screen shot of the blackboard in one of Feynman's lectures [1]; see `https://www.feynmanlectures.caltech.edu/III_01.html`.

I. THE 'ONLY MYSTERY' OF QUANTUM MECHANICS ACCORDING TO FEYNMAN

Feynman described *interference* as 'the only mystery.' He said, "We cannot make the mystery go away by "explaining" how it works. We will just tell you how it works. In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics." We reproduced one of his blackboards in Fig. 1, and there list three prinicples related to interference in quantum mechanics. We will explain these in the famous double-slit experiment and then in the Mach-Zehnder interferometer.

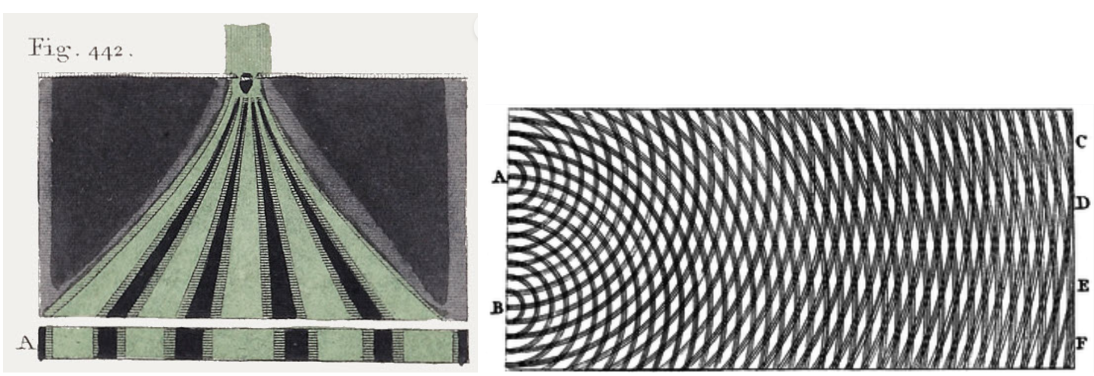

FIG. 2. Illustration of Young's double-slit experiment. Left: interference as observed by Young. Right: Young's sketch of the interference by water waves. Figures taken from `https://en.wikipedia.org/wiki/Young_interference_experiment`.

II. DOUBLE-SLIT EXPERIMENT AND THE QUANTUM MECHANICAL PERSPECTIVE

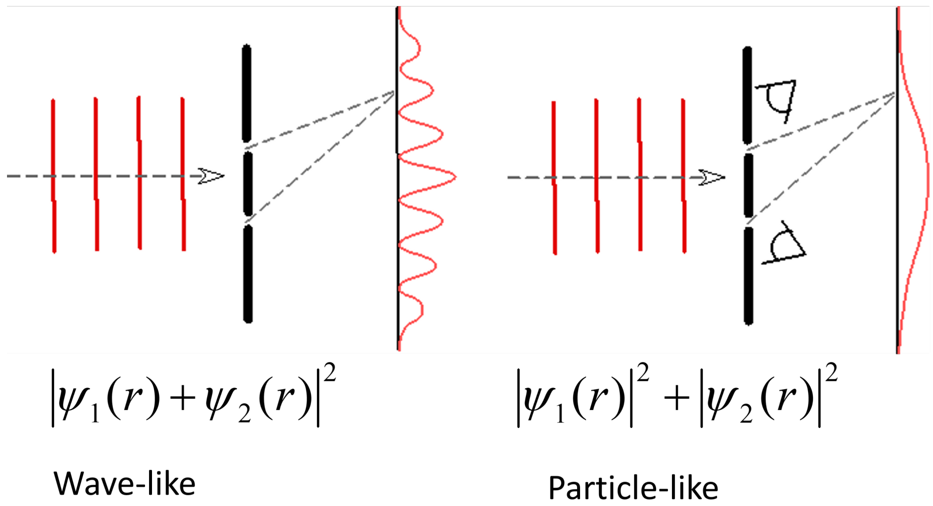

FIG. 3. Illustration of the double-slit experiment from the quantum mechanical viewpoint.

In the Young's double-slit experiment, he used a mirror to reflect the sunlight through a small hole, which was split by a paper card and an inteference pattern appeared on the screen; see Fig. 2. He drew the analogy to the interference by water waves. Interference phenomena exist in classical waves, including water, sounds, light, etc. However, it was realized that it could appear in quantum mechnical objects, such as electrons or photons (the light quantm) passing through double slits. Their quantum mechanical interference is related to the wave-particle duality, where a particle, like electrons, can exhibit wave-like phenomena [2], depending on how the experiment is conducted. Understading this helps us appreciate the 'mystery' of quantum mechanics. It is also possible that light, usually regarded as waves, can also exhibit particle behavior; for example, light in the photo-electric effect and the Compton scattering.

In quantum mechanics, we describe an object by a wave fucntion $\psi(x,t)$, which is also called a probability amplitude that can have position and time dependence. (This is the wave that de Broglie associates particles with and there is an interesting story by Felix Bloch on how Debye challenged Schrödinger to come up with a wave equation for such a wave, now known as Schrödinger's equation; see Ref. [3].) To describe the occurrence of an event, we need to square the probability amplitdue $P=|\psi|^2$. But if the event can occur via several different paths, then one needs to first sum over all the probablity amplitudes associated with all paths and then take its modulus square to infer the probability $P=|\sum_i \psi_i|^2$. In the case of the double slits, there are two possible paths, via either the top or bottom slit, and thus $P=|\psi_t+\psi_b|^2=|\psi_t|^2+|\psi_b|^2+2\mathrm{Re}(\psi_t\psi_b^*)$; see Fig. 1 and Fig. 4. It is the cross term $\psi_t\psi_b^*$ that gives rise to the interference pattern, because the cross terms changes as the location on the screen is varied.

However, the reason that the two amplitudes can interference is because that there is no way to tell which path/slit the particle (which could be an electron or a photon, the latter being the fundmantal unit of light) took to arrive at the screen. In quantum mechanics, the act of observation will perturb the system and often significantly. As illustrated in the right panel of Fig. 3, if one places a detector right behind the two slits and if a particle passes through the slit, the corresponding light will flash to signal which-path information. In this case, there will not be any inteference pattern. This is like a classical particle such as a bullet or stone, which definitely does not show interference and one in principle knows which path or hole it went through.

We want to emphasize that intrinsically, the (quantum) interference fringe pattern arises due to the particle interferes with itself, not other particles [4]. As one can perform the experiment so that the intensity of light or electron flux is so weak that there are no two or more particles in one instance of time. After enough statistics, the pattern will emerge according to Fig. 3. Somehow it seems to suggest that "the particle went through both slits at the same time," but there is no experiment that can verify this via triggering both detectors behind the slits to flash simultaneously, as there is only one particle at a time (if detector one is triggered and the other one will not). It is the lack of which-way information that gives rise to interference. It is highly recommended to read the Quantum Behavior chapter of Feynman's lectures (vol. 3).

This is not the end of the story. Even if there is a which-way information, if one can do extra things to remove the which-way information, one can in principle recover the interference pattern; see Fig. 4. Imagine that the incoming light in the experiment has a definite horizontal polarization (H), and if we simply let the light pass through the

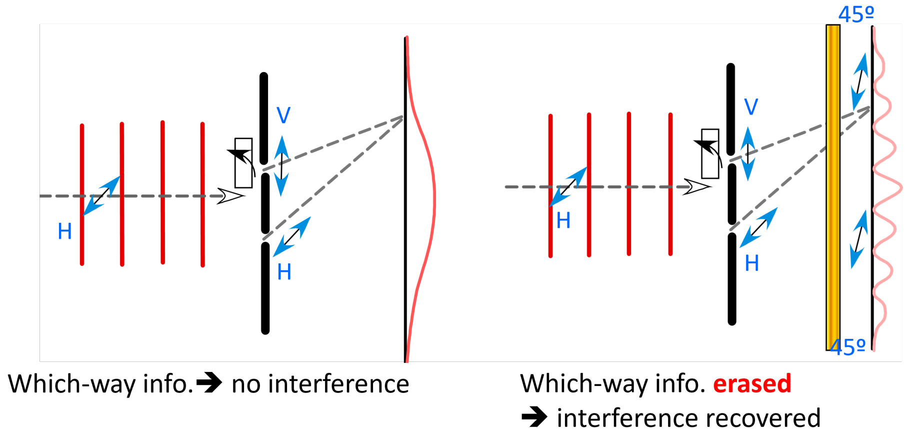

FIG. 4. Illustration of the which-way information in the double-slit experiment.

double slits, we will observe interference. However, to add a tag to which-way information, we place a waveplate right before the top slit so as to rotate the horizontal polarization (H) to the vertical polarization (V). The light coming out from the top slit has a tag V and that from the bottom slit has a tag H. H and V are completely distinguishable (or orthogonal in terms of mathematics) and constitute the which-way information. Then there will not be any interference pattern.

However, it is possible to recover the interference pattern after this. How? Wasn't the intereference pattern already gone? Yes, but if we can 'erase' the which-way information. But, how? If we place a polarizer at 45°, which allows light with polarization H or V pass with equal probability. This erases the which-way information and after that we cannot tell whether a photon arriving at the screen came from the top or the bottom slit, and there the two amplitudes can interfere.

There is a kind of delayed-choice measurement, if one decides to insert a polarizer or not after the photons have passed the double slits. The original delayed-choice measurement proposed by Wheeler is in the context of Mach-Zehnder interferometer (see, e.g., a recent experiment on this [5]).

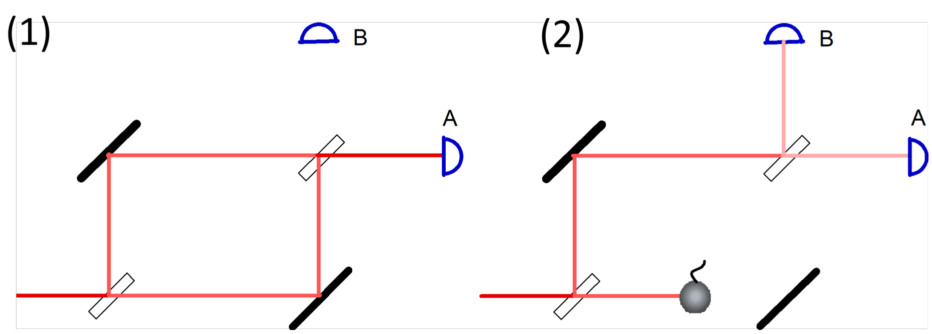

FIG. 5. Illustration of the 'interaction-free measurement' using the Mach-Zehnder interferometer.

III. MACH-ZEHNDER INTERFEROMETER AND INTERACTION-FREE MEASUREMENT

Here we will discuss an interesting thought experiment by Elitzur and Vaidman [6], whose main idea was later realized in an experiment with a Michelson interferometer [7]. We will not introduce these interferometers from the typical optical perspective, but will use the quantum behavior described above by Feynman to illustrate the bizarre feature and consequence of quantum mechanics. As illustrated in Fig. 5, there is the Mach-Zehnder (MZ) interferometer and on the left it is set up so that a photon enters from the bottom will always end up to the top port exiting to the right. (Note that one can regard the MZ interferometer as coming from the double slit experiment, where the incoming light comes from a point source, it goes through two slits and ends up at a chosen fixed point.

Sliding this end point is equivalent to changing the path length difference in the two arms of the MZ interferometer.) On the right there is a bomb in the interferometer. The introduction of the bomb is just to amplify the drama. The assumption is that the bomb is so sensitive that if a photon passes through it will definitely ignite it. The question that we would like to answer is that in the right case (1) what is the probability of the explosion? (2) What is the probability of the photon exits at port A? (3) What is the probability that it exits at port B?

First let us explain how a photon always ends up at port A? It first goes through a beam split on the bottom left, which splits it 50:50 into two separate paths. They each reflect off a mirror and rejoin at the 50:50 beam splitter on the top right. In quantum mechanics, we need to specify probability or transition amplitude. Let us assume that when a photon transmits directly through a beam splitter, the amplide is $1/\sqrt{2}$ and when it gets deflected by the beam splitter to the perpendicular direction, the amplitude is $i/\sqrt{2}$. We see that this gives the probability of transmission and of reflection being both 1/2. We will treat the mirrors as just redirecting the light beam without worrying additional (transition) amplitudes (which are π phases), i.e., assuming both mirrors give the same amplitdue and hence can be ignored.

The calculations according to Feynman's rules were illustrated in Fig. 6. You are suggested to go through the calcualtions on your own. By comparing the two cases, there is a probility of 1/4 to detector to a photon at port B if there is a bomb, whereas in the absence of a bomb, the photon never ends up at port B. This is what Elitzur and Vaidman called interaction-free measurement, as in this case, the photon has never interacted with the bomb, otherwise it would have ignited the bomb. We note that the interaction-free measurement presented above is achieved with 50% of success, but this can be extended in principle with a probability arbitrarily close to unity, using the idea of quantum Zeno effect and an experiment had demonstrated this [7].

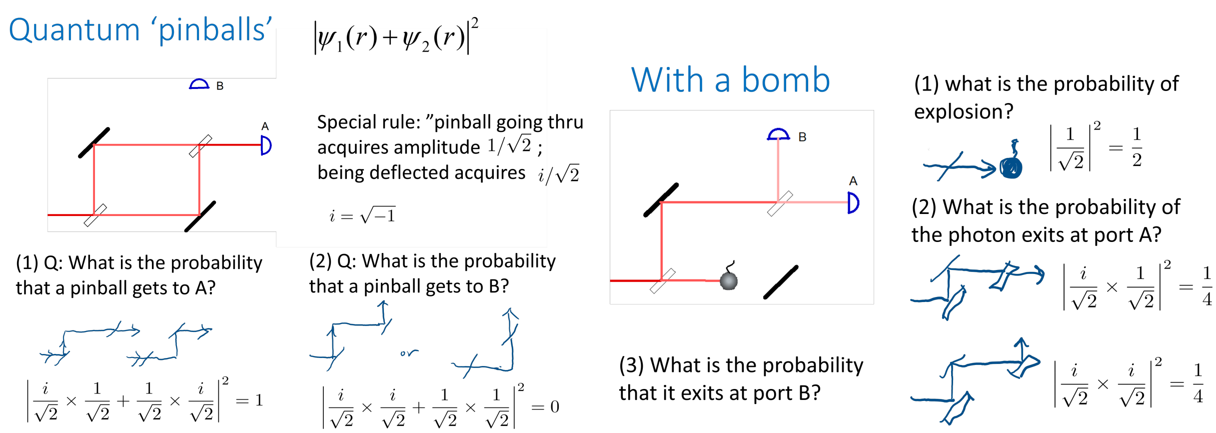

FIG. 6. Calculations of a photon undergoing the Mach-Zehnder interferometer. Left: the usual interferometer. There are two paths leading to either port A or B. Right: with a bomb. In this case, there is only one path leading to each of the three distinct outcome (hence, no interference).

You will practice the calculation to get the oscillation of probability difference at the two ports, if there is a phase shifter by a phase θ in one of the arms. This yields the usual oscillation in the classical Mach-Zehnder experiment. As a remark, in the original Wheeler's delayed-choice measurement, he imagined the delayed choice of inserting the second beam splitter or not after the photon has entered the first beam splitter. From the perspective of wave-particle duality, Englert derived an inequality between the so-called distinguishabilty $\mathcal{D}$ (related to which-way information) and visibility $\mathcal{V}$ (related to the contrast in the interference fringe): $\mathcal{D}^2+\mathcal{V}^2\leq 1$ [8], which is left for the readers to explore further. Hardy extended the consideration to a set up of two Mach-Zehnder interferometers [9], involving an electron and a positron, and a setup using photons was realized in an experiment [10]. This is also left for further exploration by the readers or possibly as a homework exercise.

The idea of Elitzur and Vaidman also led to the notion of "counterfactual computation" proposed by Mitchison and Jozsa [11], which was also later realized experimentally [12, 13]. Though, there were some debates on its efficieny [14, 15].

IV. CONCLUDING REMARKS

This unit serves as an appetizer and I hope to convince that the quantum mechanical world has pretty counter-intuitive phenomena and they are fun to think of (maybe even more fun to carry out the actual experiment). Further examples are the famous Schröding'er cat and the EPR paradox (which later led to Bell's inequality). There are a lot of interesting consequences from quantum mechanics waiting for us to explore.

Suggested reading: Feynman lectures on physics, vol. 3 [1]; see also https://www.feynmanlectures.caltech.edu/III_toc.html.

[1] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman lectures on physics, vol. 3* (Addison-Wesley, Redwood, California, 1989).

[2] De Broglie proposed the relations the wave properties from particle properties: $\lambda = h/p$, and $f = E/h$; see his thesis (translated to English) at https://fondationlouisdebroglie.org/LDB-oeuvres/De_Broglie_Kracklauer.pdf. This allows de Broglie to explain the quantization rules of Bohr and Sommerfeld by postulating that an integer number of waves should be fitted along stable orbitals.

[3] F. Bloch, Heisenberg and the early days of quantum mechanics, Physics Today **29**, 23 (1976), https://pubs.aip.org/physicstoday/article-pdf/29/12/23/8280756/23_1_online.pdf.

[4] Paul Dirac noted in his Quantum Mechanics textbook [16] that "... each photon then interferes only with itself. Interference between different photons never occurs." However, we emphasize that interference between different photons can actually occur, such as, in the famous two-photon Hong-Ou-Mandel interference [17]. There were other earlier works by Mandel and collaborators on interference from two independent lasers [18, 19]. Another example of two-photon interference is the Hanbury-Brown-Twiss effect, used in radio astronomy, which is manifested in the correlation and anti-correlation effects in the intensities received by two detectors from a source, such as a distant star.

[5] A. G. Manning, R. I. Khakimov, R. G. Dall, and A. G. Truscott, Wheeler's delayed-choice gedanken experiment with a single atom, Nature Physics **11**, 539 (2015).

[6] A. C. Elitzur and L. Vaidman, Quantum mechanical interaction-free measurements, Foundations of Physics **23**, 987 (1993).

[7] P. Kwiat, H. Weinfurter, T. Herzog, A. Zeilinger, and M. A. Kasevich, Interaction-free measurement, Phys. Rev. Lett. **74**, 4763 (1995).

[8] B.-G. Englert, Fringe visibility and which-way information: An inequality, Phys. Rev. Lett. **77**, 2154 (1996).

[9] L. Hardy, Quantum mechanics, local realistic theories, and lorentz-invariant realistic theories, Phys. Rev. Lett. **68**, 2981 (1992).

[10] W. T. M. Irvine, J. F. Hodelin, C. Simon, and D. Bouwmeester, Realization of hardy's thought experiment with photons, Phys. Rev. Lett. **95**, 030401 (2005).

[11] G. Mitchison and R. Jozsa, Counterfactual computation, Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences **457**, 1175 (2001).

[12] O. Hosten, M. T. Rakher, J. T. Barreiro, N. A. Peters, and P. G. Kwiat, Counterfactual quantum computation through quantum interrogation, Nature **439**, 949 (2006).

[13] F. Kong, C. Ju, P. Huang, P. Wang, X. Kong, F. Shi, L. Jiang, and J. Du, Experimental realization of high-efficiency counterfactual computation, Phys. Rev. Lett. **115**, 080501 (2015).

[14] G. Mitchison and R. Jozsa, The limits of counterfactual computation, arXiv preprint quant-ph/0606092 (2006).

[15] O. Hosten, M. T. Rakher, J. T. Barreiro, N. A. Peters, and P. Kwiat, Counterfactual computation revisited, arXiv preprint quant-ph/0607101 (2006).

[16] P. A. M. Dirac, *Lectures on quantum mechanics*, Vol. 2 (Courier Corporation, 2001).

[17] C.-K. Hong, Z.-Y. Ou, and L. Mandel, Measurement of subpicosecond time intervals between two photons by interference, Physical review letters **59**, 2044 (1987).

[18] G. Magyar and L. Mandel, Interference fringes produced by superposition of two independent maser light beams, Nature , 255 (1963).

[19] R. L. Pfleegor and L. Mandel, Interference of independent photon beams, Physical Review **159**, 1084 (1967).