Unit 09: Quantum computing by evolution and by measurement

Tzu-Chieh Wei
C. N. Yang Institute for Theoretical Physics and Department of Physics and Astronomy, State University of New York at Stony Brook, Stony Brook, NY 11794-3840, USA
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In this unit, we discuss other frameworks of quantum computation: such as the adiabatic and measurement-based computation.

Learning outcomes: You'll be able to understand alternative approaches for quantum computation.

I. INTRODUCTION

We have seen the standard quantum circuit model for quantum computation in an earlier unit and we have just finished Units 07 and 08 on topological codes and topological quantum computation; these are illustrated in Fig. 1. There, we also list two other frameworks: adiabatic quantum computation and measurement-based quantum computation. The former uses a time-dependent Hamiltonian so that under its time evolution, a simple, easily prepared state may be adiabatically connected to the ground state of a problem Hamiltonian. On the other hand, the latter framework operates only via local measurements, but on a prior existing quantum entangled state. In the figure, we also give names of companies that are working on specific frameworks.

II. ADIABATIC QUANTUM COMPUTATION

As stated in the introduction, we aim to drag the ground state of a simple Hamiltonian slowly to reach the ground state of a complicated one, associated with some underlying problem. This can be an optimization problem or preparation of a many-body ground state.

We then consider the following simple interpolation between the two Hamiltonians,

$$H(t) = \left(1 - \frac{t}{T}\right) H_{\text{initial}} + \frac{t}{T} H_{\text{final}}.$$

A discretized version of the time evolution is

$$|\psi(t)\rangle = e^{-iH(t_N)dt} \cdots e^{-iH(t_2)dt} e^{-iH(t_1)dt} |\psi_{\text{initial}}\rangle,\ dt = T/N,\ N \to \infty,$$

which can be formally written as a time-ordered integral,

$$|\psi(t)\rangle = \{\text{Time order}\}\, e^{-i\int_0^T dt' H(t')} |\psi_{\text{initial}}\rangle.$$

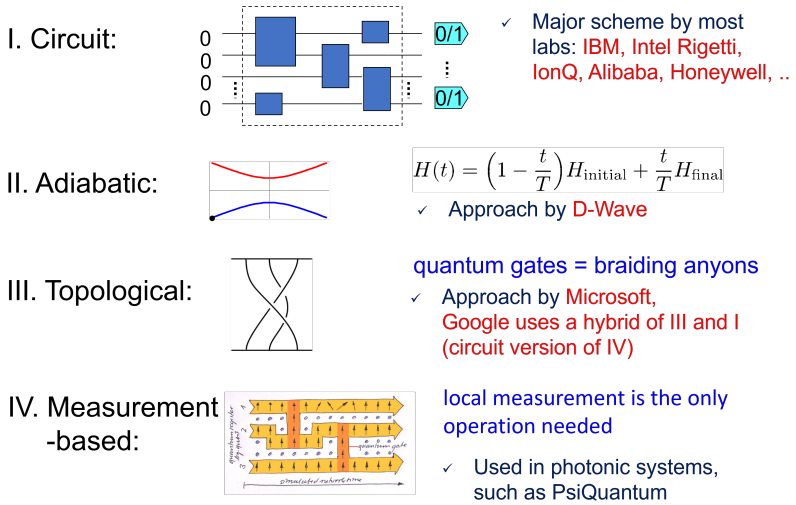


FIG. 1. Illustration of several frameworks for quantum computation.

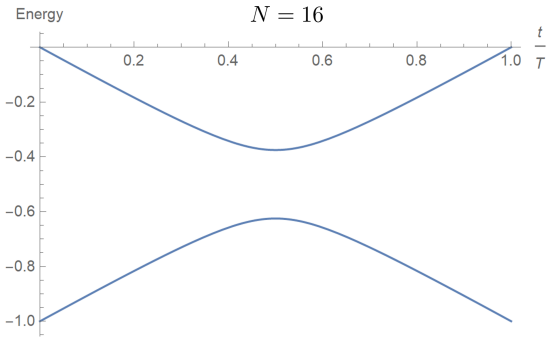


FIG. 2. Illustration of the gap $\Delta(s)$ along the Hamiltonian path.

This model of quantum computation has been shown to be (polynomially) equivalent to the standard circuit model, e.g. via the use of Feynman history state trick. See e.g. Aharonov et al. '07 [1], Lidar & Mitchell '07 [2, 3].

Before we continue with the discussion of AQC, the simplest example that illustrates the adiabatic theorem is the two-level system, where there is an off-diagonal coupling and a time-varying detuning, described by

$$i\hbar\frac{\partial}{\partial t}\left|\psi\right\rangle = -\frac{1}{2}\left(\Delta\sigma_x + vt\sigma_z\right).\left|\psi\right\rangle$$

If one starts with the system in the lowest level at $t = -\infty$, under the time-dependent evolution, there is a finite probability that the final excited will be occupied, i.e.,

$$P(t=\infty)_{\text{excited}} = e^{-2\pi\Delta^2/(4v\hbar)}.$$

This is the celebrated Landau-Zener-Stückelberg-Majorana transition [4–7], usually simply referred to as the Landau-Zener transition. The above evolution can be written in coupled differential equations and the solution is expressed in terms of a special function (i.e. the Weber function). Recently, there is an interesting paper that presents a one-line derivation of the above transition probability using Markov approximation [8]; see also the discussion in Ref. [9] regarding Majorana's solution. The implication is that if we drive the system slowly, i.e., with $v \ll \Delta^2$, then the transition probability can be made negligibly small. In fact, Michael Berry (the Berry of the Berry phase) found an alternative Hamiltonian so that the transition is exactly zero [10]. This is a useful model to keep in mind.

Now, we return to AQC. The challenge in AQC is whether one can has finite gaps (or gaps closing at most polynomially with the system size). How small should the evolution? e.g. how large T should be? It is useful to define $s = t/T$, and thus

$$\widetilde{H}(s = t/T) \equiv H(t).$$

From the adiabatic theorem, if there is a gap, then we know that we can reach very close to the final ground state,

$$|\psi(0)\rangle = |\tilde{E}_0(0)\rangle,\ |\langle\tilde{E}_0(1)|\psi(T)\rangle|^2 \to 1.$$

If we allow some error ϵ, i.e., $\||\tilde{E}_0(1)\rangle - |\psi(T)\rangle\| < \epsilon$, it was shown by Teufel '03 that

$$T \geq \frac{2}{\epsilon}\left[c_1\frac{||\dot{\tilde{H}}(0)||}{\Delta(0)^2} + c_2\frac{||\dot{\tilde{H}}(1)||}{\Delta(1)^2} + \int_0^1 ds\Big((3c_1^2 + c_1 + c_3)\frac{||\dot{\tilde{H}}||}{\Delta(s)^3} + c_2\frac{||\ddot{\tilde{H}}||}{\Delta(s)^2}\Big)\right],$$

where $\Delta(s)$ is the spectral gap along the path. A less precise, but useful to remember condition is

$$\frac{\max|\langle E_1(s)|dH(t)/dt|E_0(s)\rangle|}{\min\Delta(s)^2} \leq \epsilon.$$

A. Application to Grover's algorithm

Our discussions here follow the work by Roland and Cerf [11]. Let us assume that there is one marked item $|m\rangle$ and we have n qubits to encode $N = 2^n$ objects in total. We also define conveniently,

$$|s\rangle \equiv |++\cdots+\rangle = \frac{1}{\sqrt{N=2^n}}\sum_{x=0}^{2^n-1}|x\rangle.$$

This state is the ground state of the following Hamiltonian,

$$H_0 = I - |s\rangle\langle s|.$$

The state $|m\rangle$ corresponding to the marked item is the ground state of the following Hamiltonian,

$$H_m = I - |m\rangle\langle m|.$$

It is also convenient to define

$$\langle m|s\rangle = 1/\sqrt{N} =: a.$$

It is thus natural to define the following interpolated Hamiltonian,

$$H(t) = \Big(1 - \frac{t}{T}\Big)H_0 + \frac{t}{T}H_m = I - (1 - \frac{t}{T})|s\rangle\langle s| - \frac{t}{T}|m\rangle\langle m|.$$

The two states $|m\rangle$ and $|s\rangle$ span a two-dimensional subspace and we can write the Hamiltonian in also $|m\rangle$ and $|m^\perp\rangle$, where

$$|m^\perp\rangle = c(|s\rangle - a|m\rangle),\ c = 1/\sqrt{1-a^2},\ a = \langle m|s\rangle = 1/\sqrt{N}.$$

Then we have

$$H(t) = \begin{pmatrix} \langle m|H(t)|m\rangle & \langle m|H(t)|m^\perp\rangle \\ \langle m^\perp|H(t)|m\rangle & \langle m^\perp|H(t)|m^\perp\rangle \end{pmatrix} = I - \begin{pmatrix} (1-t/T)a^2 + t/T & (1-t/T)ca(1-a^2) \\ (1-t/T)ca(1-a^2) & (1-t/T)c^2(1-a^2)^2 \end{pmatrix}.$$

This is a 2×2 matrix and can be easily diagonalized. We found that the gap is (see Fig. 2)

$$\Delta(s = t/T) = \sqrt{1 - 4(1-1/N)(1-s)s},$$

and thus $\min_s \Delta(s) = 1/\sqrt{N}$. This seems to indicate that the time it takes to reach the final ground state with a high fidelity is proportional to N. To see this in slightly more quanitatively, we note that

$$dH(t)/dt = \frac{1}{T}(|s\rangle\langle s| - |m\rangle\langle m|) \Longrightarrow \max|\langle E_1(s)|dH(t)/dt|E_0(s)\rangle| \sim 1/T.$$

Thus, using the simplified criterion,

$$T \geq 1/(\min \Delta(s))^2/\epsilon \sim N/\epsilon,$$

showing no speedup.

However, from the behavior of the gap $\Delta(s)$, the evolution should run faster outside the minimum gap region. For example, let us take

$$ds/dt = \epsilon\Delta(s)^2 = \epsilon\Big(1 - 4(1-1/N)(1-s)s\Big).$$

We then have

$$T = \int_0^T dt = \int_0^1 \frac{ds}{\epsilon\Big(1 - 4(1-1/N)(1-s)s\Big)} = \frac{1}{\epsilon}\frac{N}{\sqrt{N-1}}\arctan\sqrt{N-1} \approx \frac{\pi}{2\epsilon}\sqrt{N} \ \text{ for large } N.$$

We note that the relation between the physical time t as a function of interpolation parameter s is (see Fig. 3)

$$t(s) = \frac{1}{2\epsilon}\frac{N}{\sqrt{N-1}}\big\{\arctan[\sqrt{N-1}(2s-1)] + \arctan\sqrt{N-1}\big\}.$$

We thus see a quadratic speedup. However, it seems in general quite tricky to find a Hamiltonian path so that the gap is sufficiently large and to find a schedule $t(s)$ in order to make efficient adiabatic quantum computation.

The quantum annealing, mostly advertised by D-Wave's systems, is in some sense a restricted adiabatic quantum computation, where the goal is to find the lowest energy state of a classical Hamiltonian, $H_{\text{classical}} = \sum_{\langle i,j\rangle} J_{jk}\sigma_z^{[i]}\sigma_z^{[j]}$. The starting point is the simple paramagnetic Hamiltonian $H_X = -\sum_i \sigma_x^{[i]}$, whose ground state is $|\psi_0\rangle = |++\cdots+\rangle$. A straightforward interpolation is $H(s) = (1-s)H_X + s\,H_{\text{classical}}$ and running some time schedule $t(s)$ to go from $s = 0$ to $s = 1$ hopefully will take $|\psi_0\rangle$ to the ground state of $H_{\text{classical}}$. However, the gap structure is complicated and it is not clear this can work efficiently. In 2010 Altshuler and collaborators wrote a paper and claimed that the physics of Anderson localization can make adiabatic quantum computation fail [12]. They used statistical methods borrowed from quantum disordered systems to analyze the gap and found that "exponentially small gaps appear close to the end of the adiabatic algorithm for large random instances of NP-complete problems."

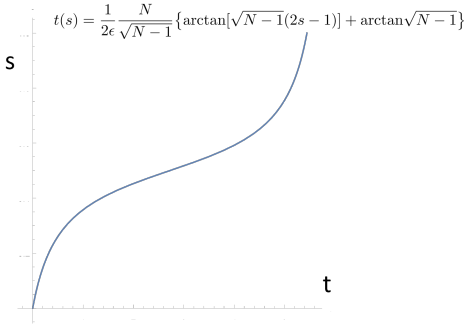


FIG. 3. Illustration of new time schedule $s(t)$.

III. MEASUREMENT-BASED QUANTUM COMPUTATION

I have written a review article on Measurement-Based Quantum Computation for the Oxford Research Encyclopedia in Physics, and the link is `https://doi.org/10.1093/acrefore/9780190871994.013.31`. At this point, I will refer the readers to this article and only cover minimal but sufficient material here. I may come back to expand this section later.

The mainstream MBQC framework was proposed by Rassendorf and Briegel in their "One-way quantum computer" [13], which uses the cluster state as the resource state. There were other perpsectives, such as, purely from measurements without any entanglement to begin with, as was done by Nielsen [14]. There was one precursory work by Gottesman and Chuang on the teleportation-based quantum computation by Gottesman and Chuang [15] and subsequently Verstraete and Cirac proposed a valence-bond picture to understand the one-way quantum computer on the cluster state [16]. This work led to the viewpoint of quantum computation in the correlation space by Gross and Eisert [17], which uses tensor networks. MBQC answers the question how entanglement can be used as a resource for quantum computation and provides connection to discussions in condensed-matter physics and phase transitions, as well as others, which we will not dwell here.

We first repeat one calculation we did in Unit 2 earlier on the gate teleportation.

A. Gate teleportation

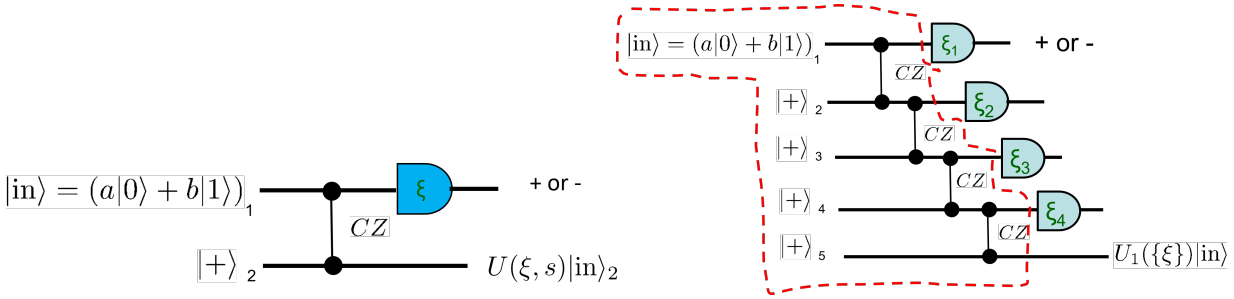


FIG. 4. Left: Illustration of the gate teleportation circuit. CZ indicates the Controlled-Z or gate and ξ is the measurement angle, defined in the main text. Right: casade of the primitive on the left by four times.

[The following material in this section was taken from my paper: "Quantum spin systems for measurement-based quantum computation" [18].

Consider an arbitrary qubit state $|\psi\rangle_1 = a|0\rangle_1 + b|1\rangle_1$, where the subscript 1 is used to label the qubit, and another qubit in the state $|+\rangle_2 = (|0\rangle_2 + |1\rangle_2)/\sqrt{2}$ state. Note that $|0\rangle$ and $|1\rangle$ are the +1 and -1 eigenstates, respectively, of the Pauli Z matrix $Z = \sigma^z$. In the following, the normalization, such as $1/\sqrt{2}$, may be dropped for ease of notation. Imagine we have a Controlled-Z gate $\text{CZ}_{mn} \equiv |0\rangle\langle 0|_m \otimes \mathbb{1}_n + |1\rangle\langle 1|_m \otimes Z_n$. Note that the CZ gate is actually symmetric, $\text{CZ}_{mn} = \text{CZ}_{nm}$, as the nontrivial action on the two qubits is only a phase shift: $|11\rangle \to -|11\rangle$. This can come from, e.g., an Ising interaction in the presence of an external field.

Applying the CZ gate to the two qubits: $\big(a|0\rangle_1+b|1\rangle_1\big)|+\rangle_2 \to |\Psi\rangle_{12} = a|0+\rangle + b|1-\rangle$, wehre $|-\rangle \equiv (|0\rangle_2 - |1\rangle_2)/\sqrt{2}$; an entanglement is thus created between qubits 1 and 2. Imagine we perform a projective measurement on the first qubit, described by the observable $\hat{O}(\xi) = \cos\xi\, X + \sin\xi\, Y$ (recall the measurement postulate in Unit 1), where $X = \sigma^x$ and $Y = \sigma^y$ are the Pauli X and Y matrices, respectively. An equivalent description of the measurement is the eigenstates of the observable $|\pm\xi\rangle \equiv (|0\rangle \pm e^{i\xi}|1\rangle)/\sqrt{2}$, with eigenvalues $\pm 1 = (-1)^s$ (or equivalently a binary variable $s = 0, 1$) to describe the measurement outcome.

Depending on the measurement outcome s on the first qubit, the second qubit is projected to (the derivation is left as an exercise or see the lecture)

$$|\psi'\rangle_2 = \langle \pm\xi|\Psi\rangle_{12} \sim H\, e^{i\xi Z/2}\, Z^s (a|0\rangle_2 + b|1\rangle_2), \tag{1}$$

where an overall phase factor is omitted. Such a procedure of (1) entangling an arbitrary input qubit $|\text{in}\rangle$ with a fixed $|+\rangle$, followed by (2) measuring the first qubit in $|\pm\xi\rangle$ basis, results in the quantum information $|\text{in}\rangle$ teleported to the second qubit, with an additional outcome-dependent unitary gate $U(\xi, s) = H\, e^{i\xi Z/2}\, Z^s$, where H is not a Hamiltonian but the so-called Hadamard gate (see Unit 1)

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{2}$$

This is referred to as the gate teleportation, schematically shown in Fig. 4. We shall see later in this course that this gate teleportation can be used to understand an alternative framework of quantum computation: the measurement-based quantum computation [19].

B. Cascading gate teleportation for one-qubit gates

We can take the output of the previous gate teleportation and insert it to the input of the next round of gate teleportation. We do this in a cascade of four such steps and by the composition we have the accumulated gate operation being

$$U_1(\{\xi\}) = \prod_{j=1}^{4} U(\xi_j, s_j) = U(\xi_4, s_4)U(\xi_3, s_3)U(\xi_2, s_2)U(\xi_1, s_1).$$

If we want to achieve an SU(2) rotation decompsed in the Euler-angle form: $R(\alpha, \beta, \gamma) = e^{-i\alpha X/2}\, e^{-i\beta Z/2}\, e^{-i\gamma X/2}$, we need to be able to "cancel out" the randomness from measurements to make the rotation deterministical. This is possible up to some byproduct operators which are simple X or Z. Take $\xi_1 = 0$ for simplicity. By using the fact that under the conjugation of the Hadamard gate, X and Z interchange, as well as that X and Z anticommute, we obtain that

$$\begin{aligned} U_1(\{\xi\}, \{s\}) \;=\; & Z^{s_1+s_3} X^{s_2+s_4}\, e^{i(-1)^{s_1+s_3}\xi_4 X/2} \\ & e^{i(-1)^{s_2}\xi_3 Z/2}\, e^{i(-1)^{s_1}\xi_2 X/2}. \end{aligned} \tag{3}$$

We see that the angles are not fixed but depend on outcomes s_i's. However, if one realizes that one has the freedom to choose the subsequent measurement angles conditioned on the prior outcomes, we can achieve an almost deterministic rotation, $R(\alpha, \beta, \gamma)$, if we take

$$\xi_2 = -(-1)^{s_1}\gamma,\ \xi_3 = -(-1)^{s_2}\beta,\ \xi_4 = -(-1)^{s_1+s_3}\alpha, \tag{4}$$

and we obtain that

$$U_1(\{\xi\}, \{s\}) = Z^{s_1+s_3} X^{s_2+s_4} R(\alpha, \beta, \gamma). \tag{5}$$

We see that there is a temporal ordering in the measurement, e.g. the measurement of qubit 2 cannot occur before that of qubit 1 ($2 > 1$) and that of 3 cannot take place before that of qubit 2 ($3 > 2$), as its actual measurement axis ξ_3 depends on s_2, the outcome of measurement on qubit 2. Moreover, we have $4 > 1$ and $4 > 3$. Collecting all the partial orderings gives the total time ordering $4 > 3 > 2 > 1$. We also have the trace of the randomness from the measurement, manifest in the byproduct operator $O_B \equiv Z^{s_1+s_3} X^{s_2+s_4}$. But if this is last step before final readout, i.e., measurement in the fixed basis defined by observable Z, then $Z^{s_1+s_3}$ only affects the phase but not the classical reading of outcome 0 or 1, which may be flipped by $X^{s_2+s_4}$ but can be corrected by hand. If this is not the end of

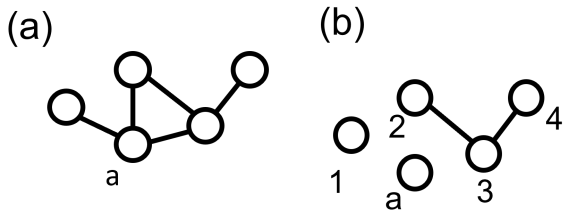


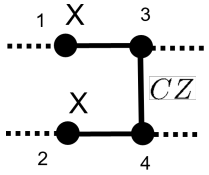
FIG. 5. Illustration of the effect of Z measurement on a 'graph' state. (a) Measuring qubit a in Z basis corresponds to (b) removing the qubit a from the graph and the remaining qubits are entangled in the structure of the remaining graph.

gate operation, we can propage this to the next desirable unitary V, e.g., by implementing instead $O_B V O_B^{-1}$ to cancel the byproduct O_B and then accumulating further byproduct operators. We repeat this until the last step when we need to read out the final outcome. The byproduct operator that can affect the readout is X, but we can correct this by flipping the outcome.

Given the ability to do rotation, the first qubit may well be initialized in the $|\text{in}\rangle = |+\rangle$ state, as other qubits. So the state $|\psi\rangle = \prod_{\langle i,j\rangle\in E} CZ_{ij}|+\cdots+\rangle$ is the cluster state, where E denotes the set of edges. Here, it is the one-dimensional line. Therefore, a one-dimensional cluster state can simulate the (discrete) time evolution of a single qubit.

C. Entangling gates

Next, we explain how one implements an entangling gate, such as CNOT or CZ. Well, the simplest answer is that, implicit in the cluster state construction, CZ gates have been applied, and we just need to route it correctly or remove them if not needed. We will consider the following structure,



Since the measurement on qubits 1 and 2 commutes with the action of CZ gate on qubits 3 and 4, let us ignore the CZ gate between qubits 3 and 4. We will perform X measurement on qubits 1 and 2, and the effect is to induce a gate $U(0,s_1)\otimes U(0,s_2)$ but the quantum information is teleporated to qubits 3 and 4 resulting in $U(0,s_1)\otimes U(0,s_2)|\text{Cin}\rangle_3\otimes|\text{Tin}\rangle_4$. Now adding the action of the CZ gate further changes the state to

$$CZ_{3,4}\,U(0,s_1)\otimes U(0,s_2)|\text{Cin}\rangle_3\otimes|\text{Tin}\rangle_4,$$

so we see that the effective action is an entangling gate

$$(CZ)\,U(0,s_1)\otimes U(0,s_2) = X^{s_1}Z^{s_2}\otimes X^{s_1}Z^{s_1}(|+\rangle\langle+|\otimes I + |-\rangle\langle-|\otimes X),$$

which is a CNOT gate but with the control in the $+/-$ basis.

There are many ways to expand the structure to make the gate implementation to be CNOT up to byproduct X and Z operators, as done in Ref. [13] and in Ref. [20]. This will give rise to some lego pieces for two-qubit gates. Together with the lego piece for the one-qubit gates, we have a universal set of gates.

D. Removing qubit by Z measurement

et consider the effect of measuring a qubit, e.g. a, in the graph state in the Z basis. As illustrated in Fig. 5, we will see that the effect is another graph state whose graph is obtained by removing the vertex a and all edges incident on it. To prove this, we first note that we can imagine building up the graph state by applying the CZ gates to between

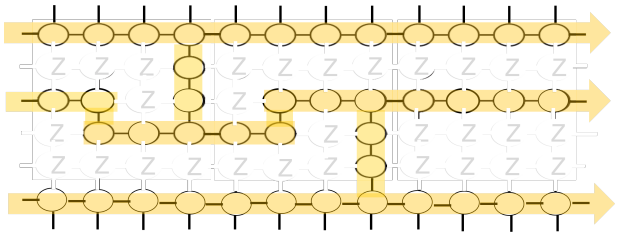


FIG. 6. Illustration of MBQC on the 2D cluster state. The yellow shaded part mimics a circuit structure.

all qubit pairs except those involving a, and we have a graph state $|\Psi_{G\setminus a}\rangle$ without the vertex a. Thus carrying the last step of remaining CZ gates to obtain the graph state $|G\rangle$ is shown as follows,

$$|\Psi_G = |0\rangle_a|\Psi_{G\setminus a}\rangle + |1\rangle_a \left(\prod_{b\in \mathrm{Nb}(a)} Z_b\right)|\Psi_{G\setminus a}\rangle, \tag{6}$$

where $\mathrm{Nb}(a)$ denotes the set of a's neighbors, i.e. $\{b|\langle a,b\rangle \in E\}$. Thus measuring in the Z basis, we either obtain $|0\rangle_a$ and the remaining qubits are projected to $|\Psi_{G\setminus a}\rangle$ or $|1\rangle_a$ wth the remaining qubits being $\left(\prod_{b\in \mathrm{Nb}(a)} Z_b\right)|\Psi_{G\setminus a}\rangle$, which is a graph state, up to the product of Pauli Z operators.

With this piece of information, we have a complete picture of performing measurement-based quantum computation on a 2D cluster states. First, we remove qubits we do not need to cut out a structure that mimic the circuit structure. Then we perform measurement in X, Y or $\cos\xi X + \sin\xi Y$ measurements (adaptively) to realize universal quantum computation.

IV. CONCLUDING REMARKS

In this unit, we have discussed other frameworks of quantum computation, such as adiabatic and measurement-based quantum computation.

It is a good time to check whether you have achieved the following Learning Outcomes:
After this Unit, you'll be able to understand alternative approaches for quantum computation.

Suggested reading: "Adiabatic Quantum Computing and Quantum Annealing" by Erica K. Grant and Travis S. Humble in Oxford Research Encyclopedia in Physics, the link is `https://doi.org/10.1093/acrefore/9780190871994.013.32`. "Measurement-Based Quantum Computation", my manuscript under review for Oxford Research Encyclopedia of Physics, the link is `https://doi.org/10.1093/acrefore/9780190871994.013.31`.

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