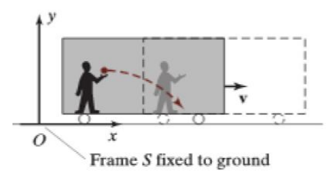
The space & time of relativity

Two great theories of first 25 years of the 20th century: (i) relativity (ii) quantum theory

(i) relativity ↓ by Einstein alone

(ii) quantum theory ↓ by many people: Bohr, Schrödinger, Heisenberg, Einstein, de Broglie, Planck, Born, ...

The theory of relativity revolutionizes our view of space & time, established by Newton. In particular reference frame:

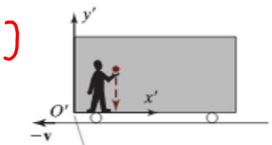


Frame S fixed to ground
(a)

Newton's three laws hold in inertial frames, which may move with constant velocity relative to each other.

e.g. $\begin{cases} t = t' & \text{(choosing same origin)} \\ x = x' + vt & \text{(}x'\text{ measured in }S'\text{ frame)} \end{cases}$ [Galilean transformation]

$\Rightarrow u = u' + v$



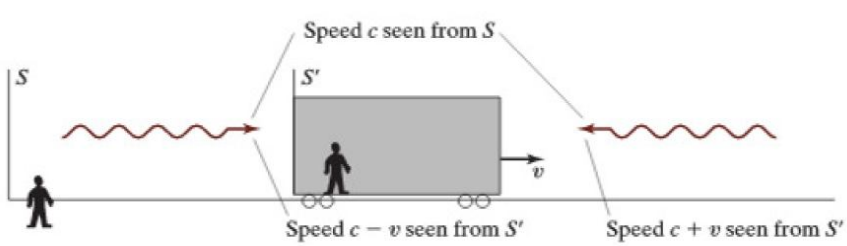
Frame S' fixed to train
(b)

Important progress leading to this: Maxwell's theory of electromagnetism:

$$\begin{cases} \nabla \cdot \vec{E} = \rho/\epsilon_0 & \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \quad \text{(Faraday's law of induction)} \\ \nabla \cdot \vec{B} = 0 & \nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \dfrac{\partial \vec{E}}{\partial t} \quad \text{[Ampere's law]} \end{cases}$$

has $c = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}} \sim 3 \times 10^8$ m/s the speed of light $\sim 1.08 \times 10^9$ km/hr

$\Rightarrow$ But Galilean transformation shows that velocity / speed is relative. $u = u' + c > 0$??



Q: what is the special reference frame such that Maxwell's equations hold & $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ predicted by the theory?

possible resolutions:

① The electromagnetic waves propagate through a special medium, called ether, and it is relative to the ether frame (the frame moving together with ether) that Maxwell's equations hold!

② later: Einstein proposed that $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ for all inertial frames!

First, let's discuss the experiment that showed ether does not exist!
(by Michelson & Morley)

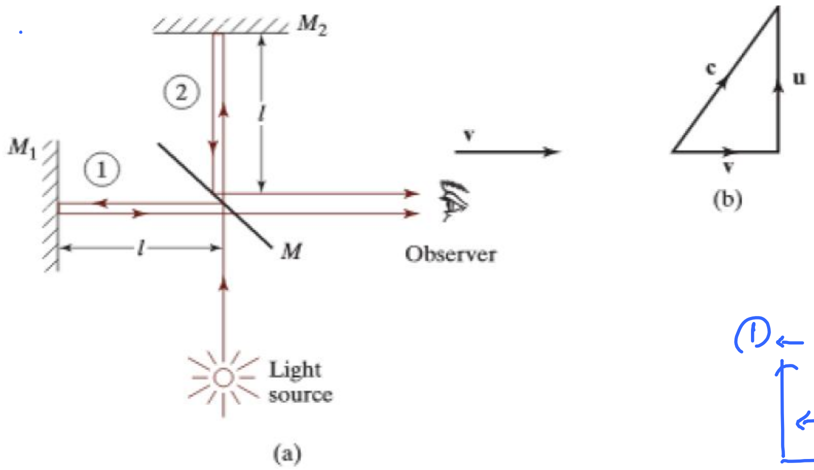
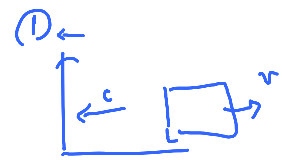


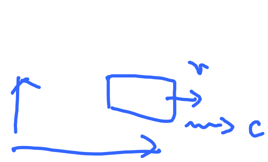
FIGURE 1.4
(a) Schematic diagram of the Michelson interferometer. M is a half-silvered mirror, M_1 and M_2 are mirrors. The vector **v** indicates the earth's velocity relative to the supposed ether frame. **(b)** The vector-addition diagram that gives the light's velocity **u**, relative to the earth, as it travels from M to M_2. The velocity **c** relative to the ether is the vector sum of **v** and **u**.



① $\quad l \quad \leftarrow : \dfrac{l}{c+v} \qquad \rightarrow : \dfrac{l}{c-v}$

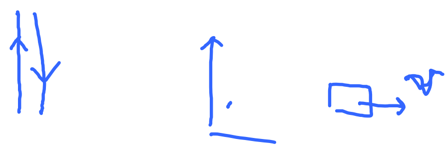
$$t_① = \frac{l}{c+v} + \frac{l}{c-v} = \frac{2cl}{c^2-v^2} = \frac{2l}{c}\frac{1}{1-\left(\frac{v}{c}\right)^2} = \frac{2l}{c}\frac{1}{1-\beta^2} \qquad \beta \equiv \frac{v}{c}$$

$\begin{cases} u_c = -c \\ u'_c = -c-v \end{cases}$



$\begin{cases} u_c = c \\ u'_c = c-v \end{cases}$

②



$$\begin{cases} u'_{cx} = u_{cx} - v = 0 & \text{as in frame } S' \text{ light moves in } y \text{ direction} \\ u'_{cy} = u_{cy} & \text{but } c^2 = u_{cx}^2 + u_{cy}^2 \text{ in frame } S: \end{cases}$$

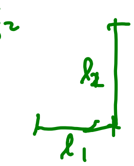
$$\Rightarrow t_② = \frac{2l}{u'_{cy}} = \frac{2l}{u_{cy}} = \frac{2l}{\sqrt{c^2-u_{cx}^2}} = \frac{2l}{\sqrt{c^2-v^2}} = \frac{2l}{c}\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{2l}{c}\frac{1}{\sqrt{1-\beta^2}}$$

$\Delta t = t_① - t_②$ $\quad$ phase shift $\Delta\varphi = \dfrac{\Delta t}{T} 2\pi = \dfrac{\Delta t}{\frac{\lambda}{c}} 2\pi = \dfrac{2\pi}{\lambda} \cdot c \cdot \dfrac{2l}{c}\left(\dfrac{1}{1-\beta^2} - \dfrac{1}{\sqrt{1-\beta^2}}\right)$

For $v \ll c$, $\beta \ll 1$. $\Delta\varphi \simeq \dfrac{2\pi}{\lambda}\dfrac{2l}{c} \cdot c\left(1+\beta^2 - \left(1+\frac{1}{2}\beta^2\right)\right) = \dfrac{2\pi l}{\lambda}\beta^2$

If $\Delta\varphi$ is $2\pi \cdot n \Rightarrow$ constructive interference
$2\pi\left(n+\frac{1}{2}\right) \Rightarrow$ destructive interference.

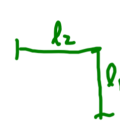
If there is a length difference. $t_① = \dfrac{2l_1}{c}\dfrac{1}{1-\beta^2}$, $t_② = \dfrac{2l_2}{c}\dfrac{1}{\sqrt{1-\beta^2}}$



$$t_① - t_② \simeq \frac{2l_1}{c}(1+\beta^2) - \frac{2l_2}{c}\left(1+\frac{1}{2}\beta^2\right) \simeq \frac{2(l_1-l_2)}{c} + \frac{2}{c}\beta^2\left(l_1 - \frac{1}{2}l_2\right)$$

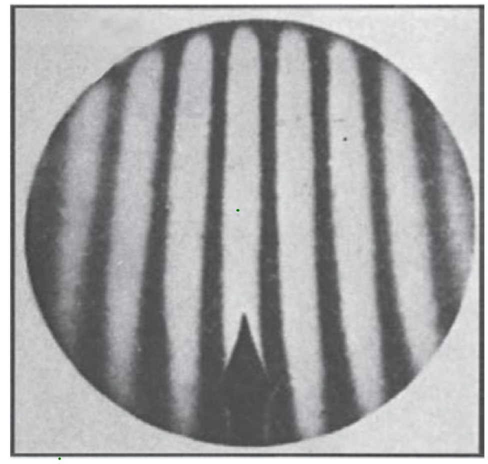
If rotated by $90°$

$$t_①' - t_②' \simeq \frac{2l_1}{c}\left(1+\frac{1}{2}\beta^2\right) - \frac{2l_2}{c}(1+\beta^2) \simeq \frac{2(l_1-l_2)}{c} - \frac{2\beta^2}{c}\left(l_2 - \frac{1}{2}l_1\right).$$



$$\Delta\varphi - \Delta\varphi' = \frac{2\pi c}{\lambda}\frac{2}{c}\beta^2\left(\frac{l_1}{2}+\frac{l_2}{2}\right) = \frac{2\pi}{\lambda}\beta^2(l_1+l_2)$$

$\Rightarrow$ There will be $\Delta N = \dfrac{\beta^2(l_1+l_2)}{\lambda}$ number of fringes shifted



From L. S. Swenson, Jr., *Invention and Discovery* 43 (Fall 1987).

In their experiment of 1887, Michelson and Morley had an arm length $l \approx 11$ m. (This was accomplished by having the light bounce back and forth between several mirrors.) The wavelength of their light was $\lambda = 590$ nm; and as we have seen, $\beta = v/c$ was expected to be of order 10^{-4}. Thus the shift should have been at least

$$\Delta N = \frac{2l\beta^2}{\lambda} \approx \frac{2 \times (11\,\text{m}) \times (10^{-4})^2}{590 \times 10^{-9}\,\text{m}} \approx 0.4 \qquad (1.11)$$

$v \sim 3 \times 10^4$ m/s earth's orbital speed around the sun.

Although they could detect a shift as small as 0.01, Michelson and Morley observed no significant shift when they rotated their interferometer.

Conclusion of Michelson-Morley experiment by Einstein leads to postulates of relativity

First Postulate of Relativity

If S is an inertial frame and if a second frame S' moves with constant velocity relative to S, then S' is also an inertial frame.

(Physical laws are the same in all inertial frames.)

Second Postulate of Relativity

In all inertial frames, light travels through the vacuum with the same speed, $c = 299{,}792{,}458$ m/s in any direction.

Two important consequences are i) time dilation and ii) length contraction

To understand these we need to examine the measurement of time and clock synchronization

We shall see that time is no longer absolute in relativity.

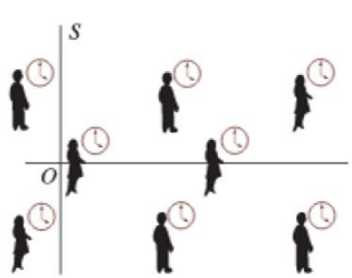
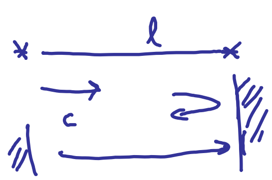


FIGURE 1.5

The chief observer at O distributes her helpers, each with an identical clock, throughout S.

To set up the clock system, one can imagine that we prepare identical clocks and "transport" them (slowly) to various locations. Alternatively, one can synchronize clocks using light as a signal. Record differences in receiving light $\Delta t = \frac{l}{c}$, given their locations are known.

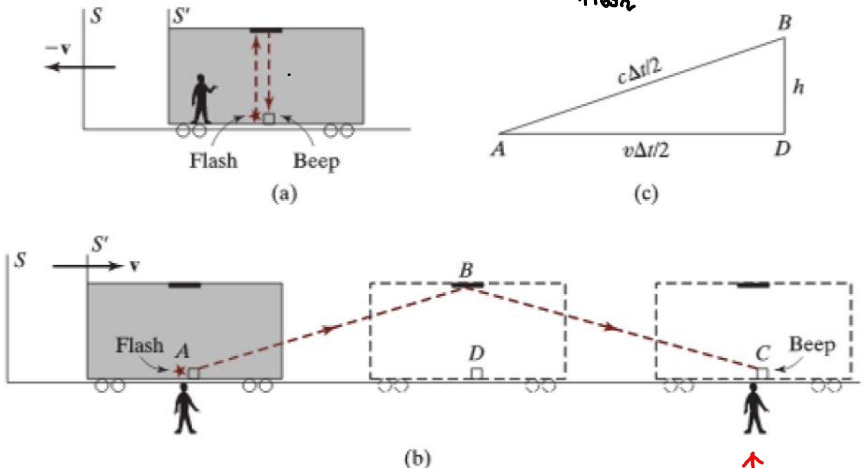


Each reference frame will be equipped with its own set of synchronized clocks.

$Oxyz$. t An event will be recorded by (x, y, z, t).

Time dilation: a thought experiment.

Observer in train (frame S') sets off a flash light at floor $\overset{h}{\uparrow\downarrow}$, bounces at mirror back to floor & beeps!



(Note here we assume height is the same as seen by both observers; see below.)

FIGURE 1.6
(a) The thought experiment as seen in the train-based frame S'. **(b)** The same experiment as seen from the ground-based frame S. Notice that two observers are needed in this frame. **(c)** The dimensions of the triangle ABD.

For S': $\Delta t' = \frac{2h}{c}$

records $\Delta t = \frac{\Delta t'}{\sqrt{1-\beta^2}}$

For observer at ground: $c\left(\frac{\Delta t}{2}\right) = \sqrt{\left(v\frac{\Delta t}{2}\right)^2 + h^2}$, $(c^2 - v^2)\left(\frac{\Delta t}{2}\right)^2 = h^2$

$$\Delta t = \frac{2h}{\sqrt{c^2 - v^2}} = \frac{2h}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \underbrace{\frac{2h}{c}}_{\Delta t'}\gamma \; ; \quad \gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} > \Delta t'$$ if $v \neq 0$ time dilation ⇒ Nothing wrong with any set of clocks.

Time is No longer absolute & depends on the reference frame!

· but $\Delta t \approx \Delta t'$ if $\frac{v}{c} \ll 1$

$$\Delta t = \frac{\Delta t'}{\sqrt{1-(0.99)^2}} \approx 7\Delta t'$$ for $\frac{v}{c} = 0.99$

$\Delta t' \equiv \Delta t_0$ proper time
denoted (measured in the frame moving with the "events")

· $\Delta t = \Delta t'$ if $v = 0$ (same reference frame, records same duration).

Evidence: ① muon decay (problem 1.27)
② atomic clocks: one flies with the airplane vs. one on ground

It was only with the advent of super-accurate atomic clocks that tests using man-made clocks became possible. The first such test was carried out in 1971. Four portable atomic clocks were synchronized with a reference clock at the U.S. Naval Observatory in Washington, D.C., and all four clocks were then flown around the world on a jet plane and returned to the Naval Observatory. The discrepancy between the reference clock and the portable clocks after their journey was predicted (using relativity) to be

(See also documentary "The illusion of time")

$$275 \pm 21 \text{ ns} \tag{1.19}$$

while the observed discrepancy (averaged over the four portable clocks) was*

$$273 \pm 7 \text{ ns} \tag{1.20}$$

1.27 •• Muons are subatomic particles that are produced several miles above the earth's surface as a result of collisions of cosmic rays (charged particles, such as protons, that enter the earth's atmosphere from space) with atoms in the atmosphere. These muons rain down more-or-less uniformly on the ground, although some of them decay on the way since the muon is unstable with a proper half-life of about 1.5 μs. (1 $\mu s = 10^{-6}$ s.) In a certain experiment a muon detector is carried in a balloon to an altitude of 2000 m, and in the course of 1 hour it registers 650 muons traveling at 0.99c toward the earth. If an identical detector remains at sea level, how many muons would you expect it to register in 1 hour? (Remember that after n half-lives the number of muons surviving from an initial sample of N_0 is $N_0/2^n$, and don't forget about time dilation.) This was essentially the method used in the first tests of time dilation, starting in the 1940's.

$t_{1/2}$: half life

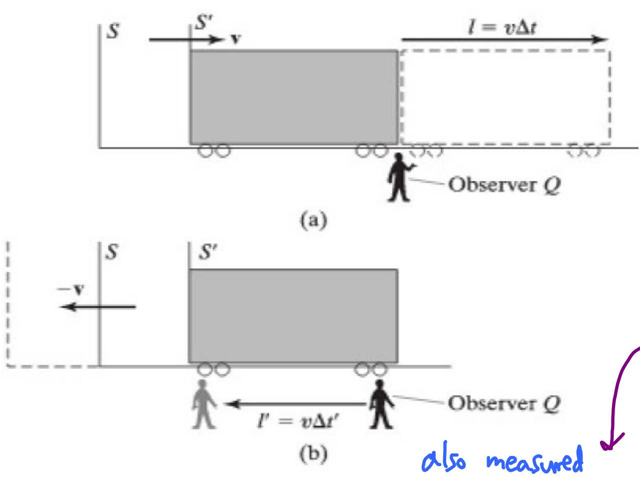
$N = N_0 \, 2^{-t/t_{1/2}}$ at $N(t = t_{1/2}) = N_0/2$

Example of pions

Length can be measured via duration.

distance = velocity × time

Length contraction:



The ground observer, the two events are at fixed location Q and lasts Δt (thus $l = v\Delta t$)

For observer on train, can directly measure the length, but also can observe how long Δt does it take for the train to pass a fixed point $\Rightarrow$ $l' = v\Delta t'$ [head & tail] [needs two clocks] one at front, one @ back

also measured directly by observer on train

But we can use time dilation. How? is it

(1) $\Delta t = \dfrac{\Delta t'}{\sqrt{1-(\frac{v}{c})^2}}$ or (2) $\Delta t' = \dfrac{\Delta t}{\sqrt{1-(\frac{v}{c})^2}}$?

Δt occurs at the same location (observer Q) $\Rightarrow$ (2)!

$l = v\Delta t = v\Delta t' \sqrt{1-(\frac{v}{c})^2} = l' \sqrt{1-\frac{v^2}{c^2}}$

$\Rightarrow$ length measured by observer fixed on ground is smaller by a factor $\sqrt{1-(\frac{v}{c})^2} = 1/\gamma$

(should also discuss the twin paradox somewhere.)

[length contraction !]

$l = \dfrac{l_0}{\gamma}$ ⇐ measured by observer in rest frame. [proper length]

Evidence: Pion experiment viewed by pions.

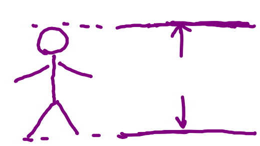
① Time dilation: from ground half-life: $t_{grnd} = t_{1/2}\,\gamma$; distance it flies $\boxed{\Delta L = v\,t_{grnd} = v\,t_{1/2}\,\gamma}$

(from π's rest frame)

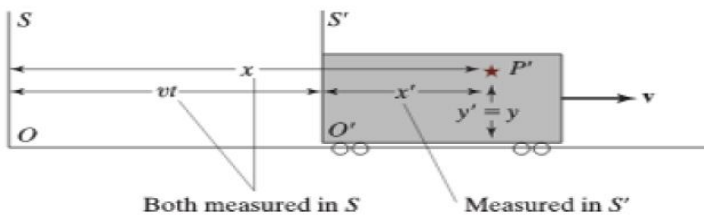
② Length contraction: half life time $t_{1/2}$: length on ground ΔL is contracted to be $\dfrac{\Delta L}{\gamma}$ $\Rightarrow$ $\boxed{v\,t_{1/2} = \dfrac{\Delta L}{\gamma}}$

Length perpendicular to relative velocity does NOT contract!

(otherwise, it will lead to contradiction.)



Lorentz transformation. application; velocity addition; Doppler effect.



According to Newton (or Galileo)

$$\begin{cases} x = x' + vt', \quad y = y', \quad z = z' \\ t = t' \end{cases}$$

Galilean transformation $\left[\begin{array}{l} x' = x - vt \\ t' = t \end{array}\right]$

Since the physics is same in both frames.

assume in relativity : $x = \gamma(x' + vt')$ and $x' = \gamma(x - vt)$

[here, we do not know γ if we do not use length contraction]

We will adjust the origin of times so that at $(t'=0, x'=0) \Rightarrow (t=0, x=0)$

Now consider light travels with speed c in both frames and

it flashes at origin $(t', x') = (t, x) = (0, 0)$.

so $\begin{cases} x' = ct' \\ x = ct \end{cases}$ must satisfy both equations $\Rightarrow \begin{cases} x = \gamma(c+v)t' \\ \quad = ct \\ x' = \gamma(c-v)t \\ \quad = ct' \end{cases}$

$$\frac{\gamma(c+v)t'}{t} = \frac{x}{t} = c = \frac{x'}{t'} = \frac{\gamma(c-v)t}{t'}$$

$$\Rightarrow \gamma^2(c+v)(c-v) = c^2 \Rightarrow \gamma^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

(consistent!)

Thus $\begin{cases} x' = \gamma(x - vt) \\ x = \gamma(x' + vt') \end{cases} \Rightarrow x' + vt' = \frac{1}{\gamma}x \Rightarrow \gamma(x - vt) + vt' = \frac{1}{\gamma}x$

$$\Rightarrow vt' = \gamma v t - \left(\gamma - \frac{1}{\gamma}\right)x$$

$$\frac{\gamma^2 - 1}{\gamma} = \frac{\frac{1}{1-\frac{v^2}{c^2}}\frac{v^2}{c^2}}{\gamma} = \gamma\frac{v^2}{c^2}$$

$$vt' = \gamma v t - \gamma\frac{v^2}{c^2}x$$

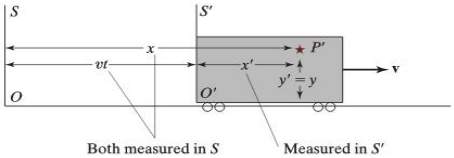
$$\Rightarrow t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$\Downarrow$$

$$\begin{cases} x' = \gamma(x - vt) \\ t' = \gamma\left(t - \frac{vx}{c^2}\right) \end{cases}$$

[Similarly $x = \gamma(x' + vt')$, $t = \gamma\left(t' + \frac{vx'}{c^2}\right)$]

The textbook uses length contraction to derive this.



As measured in S'

event x' at t'

As measured in S x, t

distance $O'P'$ measured from S

is. $x - vt \underset{\text{Contraction}}{=} \frac{1}{\gamma} x'$ $\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$

So. $x' = \gamma(x - vt)$

$\Rightarrow$ reverse sign $\Rightarrow$ reverse role S & S' $\rightarrow$ same equations as before!

$x = \gamma(x' + vt')$

$$\overbrace{\gamma(\gamma(x - vt))}^{\gamma^2 x - \gamma v t}$$

$x = \gamma(\gamma(x - vt)) + \gamma v t'$

$\gamma^2 = \frac{1}{1-\beta^2}$

$\gamma v t' = \gamma^2 v t + (1 - \gamma^2) x$

$1 - \gamma^2 = \frac{1-\beta^2-1}{1-\beta^2}$

$= \gamma^2 v t - \gamma^2 \beta^2 x$

$= -\frac{\beta^2}{1-\beta^2}$

$= -\gamma^2 \beta^2$

$\Downarrow$

$$\begin{cases} t' = \gamma\left(t - \frac{vx}{c^2}\right) \\ x' = \gamma(x - vt) \end{cases}$$

Applications (length contraction, time dilation, resolution of a paradox)

I. Derive length contraction using Lorentz transformation

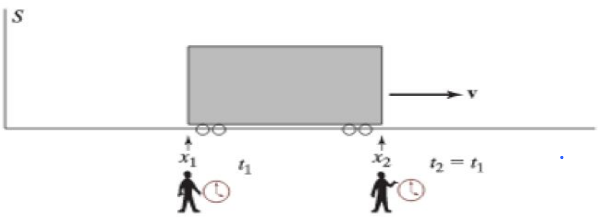


FIGURE 1.10
If the two observers measure x_1 and x_2 at the same time ($t_1 = t_2$), then $l = x_2 - x_1$.

Event	Description	Coordinates in S
1	Back of train passes first observer	x_1, t_1
2	Front of train passes second observer	$x_2, t_2 = t_1$

$$x_1' = \gamma(x_1 - v t_1) \qquad t_1' = \gamma\left(t_1 - \frac{v x_1}{c^2}\right)$$

$$x_2' = \gamma(x_2 - v t_1) \qquad t_2' = \gamma\left(t_1 - \frac{v x_2}{c^2}\right)$$

[length in S' is $x_2' - x_1'$] $\Rightarrow l' = x_2' - x_1' = \gamma(x_2 - x_1) = \gamma l$

i.e. $l = \dfrac{l'}{\gamma}$

II. Re-derive time dilation

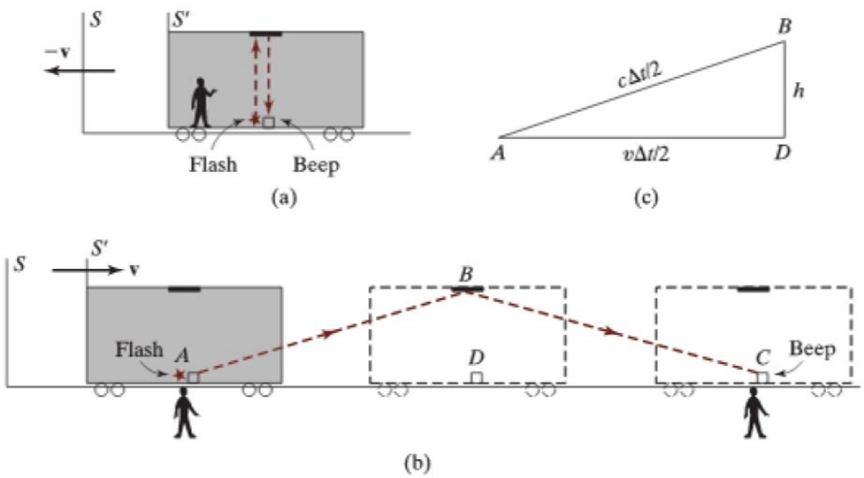


FIGURE 1.6
(a) The thought experiment as seen in the train-based frame S'. **(b)** The same experiment as seen from the ground-based frame S. Notice that two observers are needed in this frame. **(c)** The dimensions of the triangle ABD.

Flash (x', t_1') beep (x', t_2')

$$\Downarrow \qquad\qquad \Downarrow$$

$$x_1 = \gamma(x' + v t_1') \qquad x_2 = \gamma(x' + v t_2')$$

$$t_1 = \gamma\left(t_1' + \frac{v x'}{c^2}\right) \qquad t_2 = \gamma\left(t_2' + \frac{v x'}{c^2}\right)$$

$$\Rightarrow \Delta t = t_2 - t_1 = \gamma(t_2' - t_1') = \gamma \Delta t'$$

time dilation.

III. A paradox.

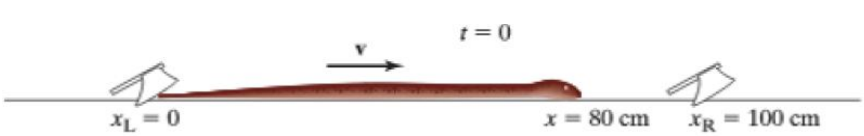


FIGURE 1.11
As seen in the boy's frame S, the two hatchets bounce simultaneously (at $t = 0$) 100 cm apart. Since the snake is 80 cm long, it escapes injury.

Example 1.6

Q: who is correct? why is the other wrong?

A relativistic snake of proper length 100 cm is moving at speed $v = 0.6c$ to the right across a table. A mischievous boy, wishing to tease the snake, holds two hatchets 100 cm apart and plans to bounce them simultaneously on the table so that the left hatchet lands immediately behind the snake's tail. The boy argues as follows: "The snake is moving with $\beta = 0.6$. Therefore, its length is contracted by a factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.36}} = \frac{5}{4}$$

and its length (as measured in my rest frame) is 80 cm. This implies that the right hatchet will fall 20 cm in front of the snake, and the snake will be unharmed." (The boy's view of the experiment is shown in Fig. 1.11.) On the other hand, the snake argues thus: "The hatchets are approaching me with $\beta = 0.6$, and the distance between them is contracted to 80 cm. Since I am 100 cm long, I will be cut in pieces when they fall." Use the Lorentz transformation to resolve this apparent paradox.

Reference frames

$S \xrightarrow{v} S'$

Snake @ rest in S': tail at $(t'=0,\ x'_t=0)$
head at $(t'=0,\ x'_h = 100\text{ cm})$

hatchets at rest in S:
left $x=0$
right $x = 80\text{ cm}$

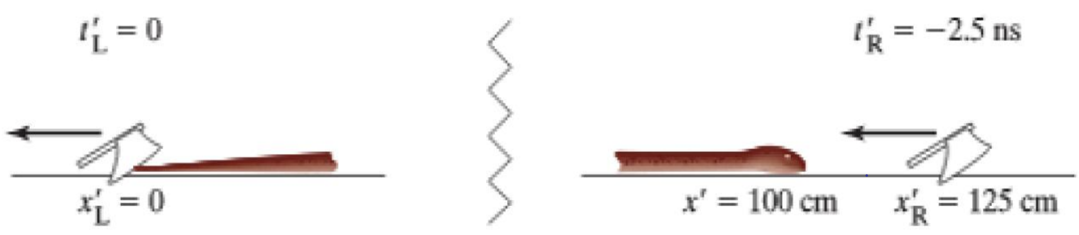
boy launches left hatchets when tail at $x=0$ (set this $t=0$)

Relative to S frame, at $t=0$, tail at $x=0$, head at 80 cm correct?

yes $x' = \gamma(x - vt) = \gamma \cdot 80 = 100\text{ cm}$ but $t' = \gamma\left(0 - \frac{v \cdot 80}{c^2}\right) = -\frac{5}{4} \cdot 0.6\,\frac{80}{c} = \frac{-60\text{ cm}}{c}$

so right hatchet was place at $t=0$ at $x = 100\text{ cm}$, which corresponds to
$x' = \gamma(100 - v \cdot 0) = 125\text{ cm}$, $t' = \gamma\left(0 - \frac{v\,100}{c^2}\right) = -\frac{5}{4}\,0.6\,\frac{100}{c} = -\frac{75\text{ cm}}{c}$

From snake's viewpoint left hatchet was released at $x'=0,\ t'=0$, when its head is at $x' = 100\text{ cm},\ t'=0$; but right hatchet was already released earlier at $x' = 125\text{ cm}$ $t' = -\frac{75\text{ cm}}{c}$ (Snake is unharmed)
$\approx -2{,}5\text{ ns}$



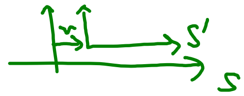
(a) (b)

Q: what if boy release right hatchet at $x = 80\text{ cm}$, $t=0$?
what location of space time does the snake see the event?

right hatchet: $x' = \gamma(80 - v \cdot t) = 100\text{ cm}$, $t' = \gamma\left(0 - \frac{v \cdot 80}{c^2}\right) = -\boxed{0.6} \times \frac{100\text{ cm}}{c} = -2\text{ ns}$

(tail was at $x'=0$, $t'=0$)
left hatchet $x=0, t=0$

Velocity addition



$x' = \gamma(x - vt)$ How would a velocity u measured in S relate to u' measured in S'?

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$ Q: is it just $u' = u - v$? We know it cannot be correct at least for light.

So what is the correct formula?

$$u_x' = \frac{dx'}{dt'} \qquad u_x = \frac{dx}{dt}$$

$$\Rightarrow u_x' = \frac{\gamma(dx - v\,dt)}{\gamma\left(dt - \frac{v}{c^2}dx\right)} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{u_x - v}{1 - \frac{v u_x}{c^2}} \quad \text{for } x \text{ direction}$$

[The reverse relations are obtained by setting v to $-v$]

What about u_y' & u_y? Since $y' = y$

$$u_y' = \frac{dy'}{dt'} = \frac{dy}{\gamma\left(dt - \frac{v}{c^2}dx\right)} = \frac{dy/dt}{\gamma\left(1 - \frac{v}{c^2}u_x\right)} = \frac{u_y}{\gamma\left(1 - \frac{v u_x}{c^2}\right)}$$

The relations reduce to, for small u_x i.e. $v/c \ll 1$,

$$u_x' \approx u_x - v, \quad u_y' \approx u_y \quad \text{as } \gamma \approx 1. \ (\beta \approx 0)$$

The rocket of Example 1.7 shoots forward a signal (for example, a pulse of light) with speed c relative to the rocket. What is the signal's speed relative to the earth? First, from the 2nd postulate, we should have $u = c$. But we can verify it using "addition"

In this case $u' = c$. Thus according to (1.45)

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{c + v}{1 + v/c} = c \tag{1.46}$$

Exercise (optional)

1.47 •• Using the velocity-addition formula, one can prove the following important theorem: If a body's speed u relative to an inertial frame S is less than c, its speed u' relative to any other inertial frame S' is also less than c. In this problem you will prove this result for the case that all velocities are in the x direction.

Suppose that S' is moving along the x axis of frame S with speed v. Suppose that a body is traveling along the x axis with velocity u relative to S. (We can let u be positive or negative, so that the body can be traveling either way.) **(a)** Write down the body's velocity u' relative to S'. For a fixed positive v (less than c, of course), sketch a graph of u' as a function of u in the range $-c < u < c$. **(b)** Hence prove that for any u with $-c < u < c$, it is necessarily true that $-c < u' < c$.

Doppler effect

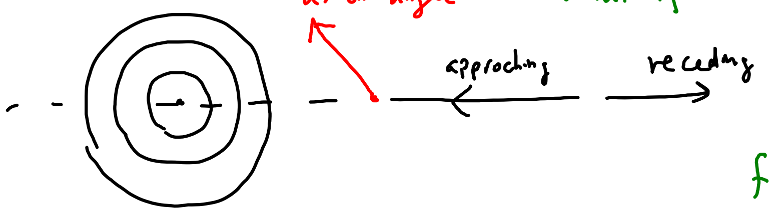
Unlike the sound waves in the non-relativistic case (e.g. Giancoli ch.)
the Doppler effect for light is modified

We don't have time for derivation, will just quote the results; (which is simpler than that of sounds)

$$f_{obs} = \sqrt{\frac{1+\beta}{1-\beta}}\, f_{sce} \qquad \text{(approaching)} \qquad (1.53)$$

$$f_{obs} = \sqrt{\frac{1-\beta}{1+\beta}}\, f_{sce} \qquad \text{(receding)} \qquad (1.54)$$

at an angle? What if the motion is at an angle?



(see prob 1.53)

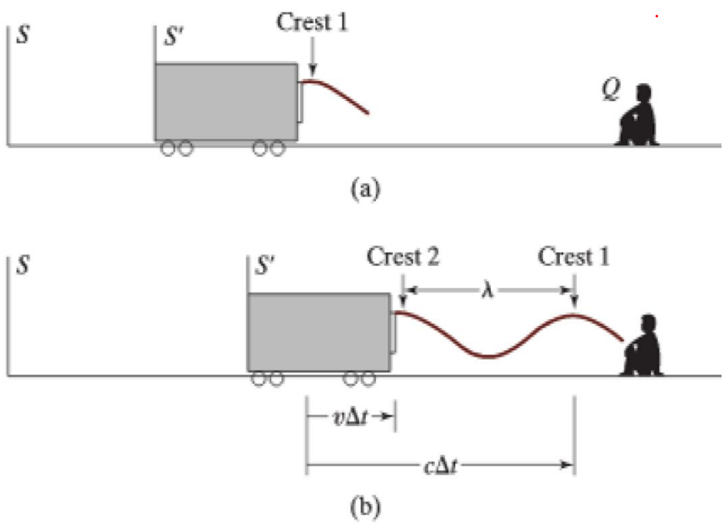
$f_{sce} = \frac{1}{\Delta t'}$ $\Delta t'$ between crests

$f_{obs} = \frac{c}{\lambda}$; $\lambda = c\Delta t - v\Delta t$

$\Delta t = \gamma \Delta t'$

$= \frac{c}{(c-v)\Delta t} = \frac{1}{(1-\beta)}\frac{1}{\gamma}\frac{1}{\Delta t'} = \frac{f_{sce}}{1-\beta}\sqrt{1-\beta^2}$

$= f_{sce}\sqrt{\frac{1+\beta}{1-\beta}}$



Q: what if there is a mirror and the light bounces back?

Hubble discovered that galaxies are all redshifted → the universe is expanding.