Relativistic mechanics

Galilean transf $\xrightarrow{\text{replaced by}}$ Lorentz transf.

Newtonian mechanics $\longrightarrow$ relativistic mechanics (what is it?)

time, position $\Rightarrow$ velocity, acceleration $\longrightarrow$ same def. (using appropriate ref. frame)

What about notions for mass? momentum? energy? how is force related to momentum change?

What do we expect of relativistic mechanics?

a. valid in all inertial frames (invariant under Lorentz transf.)

b. reduces to non-relativistic one for $v/c \ll 1$

c. agrees w. experiment.

Mass: (b) $\Rightarrow$ same as classical $m = F/a$ valid for $v/c \ll 1$ (cannot accelerate to large v)

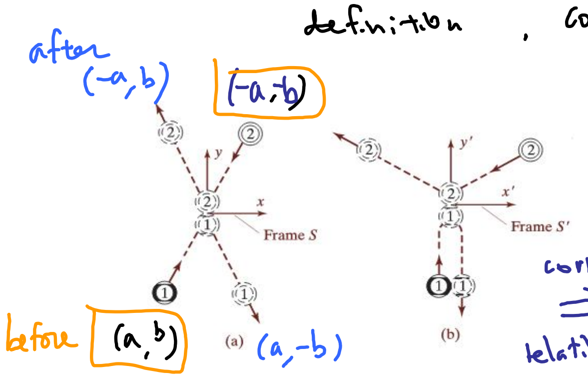
$\Rightarrow$ implies we should start from $v=0$. to apply $m = F/a$

i.e. bring object to rest before measuring mass [rest mass, proper mass]

(this implies (a) will hold)

momentum:

Newtonian $\vec{p} = m\vec{u} = m\frac{d\vec{r}}{dt} \longrightarrow$ it turns out with such a definition, conservation of (total) momentum may not hold. $\sum_i \vec{p}_i = \text{const}$?



$\sum_i m_i \vec{u}_i$ conserved in frame S, but NOT in S'

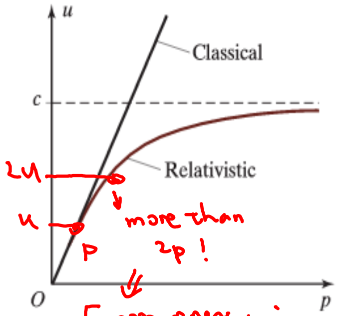
see prob. 2.5

correct $\Longrightarrow$ relativistic

$$\boxed{\vec{p} = m\frac{d\vec{r}}{dt_0}} \longleftarrow \text{use proper time}$$

$$= m\left(\frac{dt}{dt_0}\right)\frac{d\vec{r}}{dt}$$

$$= m\gamma\vec{u}, \quad \text{where } \gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$



[more energy is required to bump up u]

For an object with nonzero mass, its momentum will need to be very large when traveling close to speed of light!

For classical force $\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$. It turns out that $\vec{F} = \frac{d\vec{p}}{dt}$ is the correct definition in relativity!

If we use this definition of force and consider energy increase,

$$dE = \vec{F} \cdot d\vec{r} = \frac{d\vec{p}}{dt} \cdot d\vec{r} = d\vec{p} \cdot \vec{u} = d(m\gamma(u)\vec{u}) \cdot \vec{u}$$

Let integrate along a straight line $\vec{0} \to \vec{u}$

$$\int dE = \int d(m\gamma(u)u) \cdot u = m\gamma(u)u\,u\Big| - \int m\gamma(u)u\,du$$

$$= \frac{mu^2}{\sqrt{1-u^2/c^2}}\Bigg|_0^u - m\int_0^u du \frac{u}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$E - E_0 = \frac{mu^2}{\sqrt{1-\frac{u^2}{c^2}}} - m\int_0^u \frac{\frac{1}{2}du^2}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\frac{1}{2}\frac{dx}{\sqrt{1-\frac{x}{c^2}}} = -d\sqrt{1-\frac{x}{c^2}}\;c^2$$

$$= \frac{mu^2}{\sqrt{1-\frac{u^2}{c^2}}} + mc^2\sqrt{1-\frac{u^2}{c^2}} - mc^2$$

$$= m\frac{\cancel{u^2}+c^2-\cancel{u^2}}{\sqrt{1-\frac{u^2}{c^2}}} - mc^2 \Rightarrow E(u) - E(0) = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} - mc^2$$

we identify $E(u) = \dfrac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma mc^2 = \dfrac{dt}{dt_0}mc^2$ (Prob 2.10)

[in $\boxed{\text{Prob 2.14}}$ it is proved that such energy expression gives conservation of energy in all inertial frames.]

When $u=0$, $E = mc^2$ (rest energy)

example 2.2. $m = 1\text{kg}$. $E = mc^2 = 1\text{kg}\cdot(3\times10^8\ \text{m/s})^2 = 9\times10^{16}$ joules

In practice we can only convert a small fraction! ≈ energy of power plant in one year!

Do we recover classical limit if $u/c \ll 1$?

$$E(u) \approx mc^2 \left(1 + \frac{1}{2}\frac{u^2}{c^2}\right) \approx mc^2 + \underbrace{\frac{1}{2}mu^2}_{\text{classical kinetic energy}}$$ yes ! (mc^2 is just a constant)

$\Downarrow$

Kinetic energy $K \equiv E - mc^2 = (\gamma - 1)\, mc^2$

Example 2.3 Collision

Before (m_1 & m_2) After

Assume conservation of momentum & energy (elastic collision)

Classical (Newtonian physics): $u_3 = \frac{m_1 - m_2}{m_1 + m_2} u_1$, $u_4 = \frac{2m_1}{m_1 + m_2} u_1$

Relativistic :

momentum : $\gamma_1(u_1)\, m_1 u_1 + 0 = \gamma_3(u_3)\, m_1 u_3 + \gamma_4(u_4)\, m_2 u_4$

energy $\gamma_1(u_1) m_1 c^2 + m_2 c^2 = \gamma_3(u_3)\, m_1 c^2 + \gamma_4(u_4)\, m_2 c^2$

Algebra is quite messy and we obtain

$$u_3 = \frac{m_1^2 - m_2^2}{m_1^2 + m_2^2 + 2m_1 m_2 \sqrt{1 - u_1^2/c^2}}\, u_1$$ $\longrightarrow$ reduces to classical one if $u_1/c \ll 1$

Two useful relations

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1 - u^2/c^2}} \qquad E = \frac{mc^2}{\sqrt{1 - u^2/c^2}}$$

(photon) [neutrino has mass!]

$$\Rightarrow \boxed{\frac{\vec{p}}{E} = \frac{\vec{u}}{c^2} = \frac{1}{c}\vec{\beta}}$$

* valid even for massless particle ; $m = 0$.

$E^2 = p^2 c^2$

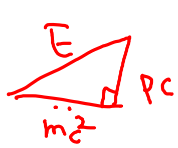
i.e. $|\vec{u}| = \frac{c^2 |\vec{p}|}{E} = c$

$\Downarrow$

massless particles travel at speed of light!

via ①

$$E^2 = \frac{m^2 c^4}{1 - \frac{u^2}{c^2}} = m^2 c^4 \left(1 + \frac{u^2/c^2}{1 - \frac{u^2}{c^2}}\right) \Rightarrow \boxed{E^2 = (pc)^2 + (mc^2)^2}$$



$\Rightarrow m \neq 0 \rightarrow u < c$!

Algebra: (skip this & next page if you don't need to see detailed steps.)

for the collision problem:

starting pt
$$\begin{cases} m_1\sqrt{\gamma_1^2-1} + 0 = m_1\sqrt{\gamma_3^2-1} + m_2\sqrt{\gamma_4^2-1} \\ m_1\gamma_1 + m_2 = m_1\gamma_3 + m_2\gamma_4 \end{cases}$$

$$\left(m_1\sqrt{\gamma_1^2-1} - m_1\sqrt{\gamma_3^2-1}\right)^2 = m_2^2(\gamma_4^2-1) = m_2^2\left(\left(\frac{m_1\gamma_1+m_2-m_1\gamma_3}{m_2}\right)^2-1\right)$$
$$= (m_1\gamma_1+m_2-m_1\gamma_3)^2 - m_2^2$$

$$m_1^2(\gamma_1^2-1) + m_1^2(\gamma_3^2-1)$$
$$-2m_1m_1\sqrt{\gamma_1^2-1}\sqrt{\gamma_3^2-1} = (m_1\gamma_1+m_2-m_1\gamma_3)^2 - m_2^2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \gamma^2 = \frac{1}{1-\beta^2} \Rightarrow \gamma^2-1 = \frac{\beta^2}{1-\beta^2} \qquad \frac{\gamma^2-1}{\gamma^2} = \beta^2$$

$$\cancel{m_1^2}\left(\cancel{-2}+\cancel{\gamma_1^2}+\cancel{\gamma_3^2} - 2\sqrt{\gamma_1^2-1}\sqrt{\gamma_3^2-1}\right) = \cancel{m_1^2}(\gamma_1-\gamma_3)^2 + \cancel{m_2^2} + 2\cancel{m_1^2}(\gamma_1-\gamma_3)\frac{m_2}{m_1} - \cancel{m_2^2}$$
$$\cancel{\gamma_1^2-2\gamma_1\gamma_3+\gamma_3^2}$$

$$+\cancel{1} + \sqrt{\gamma_1^2-1}\sqrt{\gamma_3^2-1} = +\gamma_1\gamma_3 - \frac{m_2}{m_1}(\gamma_1-\gamma_3)$$

$$(\gamma_1^2-1)(\gamma_3^2-1) = \left[\left(-1-\frac{m_2}{m_1}\gamma_1\right) + \left(\gamma_1+\frac{m_2}{m_1}\right)\gamma_3\right]^2$$
$$= \left(\gamma_1+\frac{m_2}{m_1}\right)^2\gamma_3^2 - 2\left(1+\frac{m_2}{m_1}\gamma_1\right)\left(\gamma_1+\frac{m_2}{m_1}\right)\gamma_3 + \left(1+\frac{m_2}{m_1}\gamma_1\right)^2$$

Convenient to define:

$$\rho \equiv \frac{m_2}{m_1} \qquad \left[(\gamma_1+\rho)^2-(\gamma_1^2-1)\right]\gamma_3^2 - 2(1+\rho\gamma_1)(\gamma_1+\rho)\gamma_3 + (1+\rho\gamma_1)^2 + (\gamma_1^2-1) = 0$$

$$\Rightarrow (2\gamma_1\rho+\rho^2+1)\gamma_3^2 - 2(1+\rho\gamma_1)(\gamma_1+\rho)\gamma_3 + \rho^2\gamma_1^2 + 2\rho\gamma_1 + \gamma_1^2 = 0$$

Two solutions:

$$\gamma_3 = \frac{\cancel{2}(1+\rho\gamma_1)(\gamma_1+\rho) \pm \sqrt{(1+\rho\gamma_1)^2(\gamma_1+\rho)^2 - (2\gamma_1\rho+\rho^2+1)(\rho^2\gamma_1^2+2\rho\gamma_1+\gamma_1^2)}}{\cancel{2}(2\gamma_1\rho+\rho^2+1)}$$

algebra

$$(1+2\rho\gamma_1+\rho^2\gamma_1^2)(\gamma_1^2+2\rho\gamma_1+\rho^2) - (2\gamma_1\rho+\rho^2+1)(\rho^2\gamma_1^2+2\rho\gamma_1+\gamma_1^2)$$

$$= (2\rho\gamma_1+\rho^2\gamma_1^2)(\gamma_1^2-1) + (2\rho\gamma_1+\rho^2)(1-\gamma_1^2)$$

$$= (\cancel{2\rho\gamma_1}+\rho^2\gamma_1^2-\cancel{2\rho\gamma_1}-\rho^2)(\gamma_1^2-1)$$

$$= \rho^2(\gamma_1^2-1)^2$$

So $$\gamma_3 = \frac{(1+\rho\gamma_1)(\gamma_1+\rho) \pm \rho(\gamma_1^2-1)}{2\gamma_1\rho+\rho^2+1}$$

gives $\gamma_3 = \gamma_1$ (i.e. $u_3 = u_1$, $u_4 = 0$ initial condition) or

$$\gamma_3 = \frac{\gamma_1(\rho^2+1)+2\rho}{2\rho\gamma_1+(\rho^2+1)}$$

$$\Rightarrow \beta_3^2 = 1 - \frac{(2\rho\gamma_1+\rho^2+1)^2}{(\gamma_1(\rho^2+1)+2\rho)^2}$$

$$= \frac{(\gamma_1^2-1)(\rho^2-1)^2}{(\gamma_1(\rho^2+1)+2\rho)^2}$$

sign

$$\beta_3 = \frac{\sqrt{\gamma_1^2-1}\,(\rho^2-1)}{\gamma_1(\rho^2+1)+2\rho}$$

Sign should be $m_1^2-m_2^2$

as if $m_1 > m_2$, $u_1 > 0$

$m_1 < m_2$, $u_1 < 0$ recoil

$$= \frac{\beta_1\cancel{\gamma_1}(m_1^2-m_2^2)}{\cancel{\gamma_1}(m_2^2+m_1^2)+2m_1m_2/\gamma_1} \qquad \gamma_1^2 = \frac{1}{1-\beta_1^2} \qquad \gamma_1^2-1 = \frac{\beta_1^2}{1-\beta_1^2}$$

$$\boxed{\beta_3 = \frac{\beta_1(m_1^2-m_2^2)}{m_1^2+m_2^2+2m_1m_2\sqrt{1-\beta_1^2}}}$$

Hence $$u_3 = \frac{m_1^2-m_2^2}{m_1^2+m_2^2+2m_1m_2\sqrt{1-u_1^2/c^2}}\,u_1$$ ✗✗

Example 2.4 & 2.5 using eV & MeV as energy units

Energy unit eV. In addition to joule = NT$\cdot$ meter = $kg \cdot \frac{m^2}{s^2}$

we can also use electron volt $eV = (1.6 \times 10^{-19}\ \text{coulomb}) \times \text{volt}$

$= 1.6 \times 10^{-19}$ Joule

$MeV = 10^6\ eV = 1.6 \times 10^{-13}\ J$

$GeV = 10^9\ eV = 1.6 \times 10^{-10}\ J$

Then mass can be measured in unit of $eV/c^2 = \frac{1.6 \times 10^{-19}\ J}{(3 \times 10^8)^2\ (\frac{m}{s})^2} = 1.778 \times 10^{-36}$ kg

Example 2.4

electron mass $m_e = 9.109 \times 10^{-31}$ kg $= \frac{9.109 \times 10^{-31}}{1.778 \times 10^{-36}} \frac{eV}{c^2} = 5.11 \times 10^5 \frac{eV}{c}$
(rest mass)

$= 0.511\ MeV/c^2$

Physicists often just say electron's mass is 0.511 MeV

(with $1/c^2$ implicit)

In this unit proton mass $= 938.27\ MeV/c^2$

neutron mass $= 939.57\ MeV/c^2$

Example 2.5

mass unit MeV/c^2 then momentum unit is MeV/c

If electron is moving with total energy $E = 1.3\ MeV$

momentum $p = ?$ and speed $u = ?$

$$E = \sqrt{m_e^2 c^4 + p^2 c^2} \qquad p = \sqrt{E^2 - (m_e c^2)^2}\ /c = \frac{\sqrt{(1.3)^2 - (0.5)^2}\ MeV}{c} = 1.2\ \frac{MeV}{c}$$

$$\frac{u}{c} = \beta = \frac{pc}{E} = \frac{1.2}{1.3} = 0.92$$

$\Longrightarrow u = 0.92c$

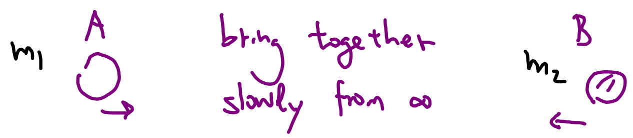
$$= \frac{1.2 \times 10^6 \times 1.6 \times 10^{-19}\ J}{3 \times 10^8\ m/s}$$

$$\simeq 6 \times 10^{-22}\ kg \cdot m/s$$

Mass $\Rightarrow$ Energy

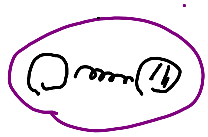
- process violates classical conservation of mass

but logically necessary in relativity.



$\Rightarrow$ could have repulsive or attractive when close

e.g. repulsive



rest mass M

: need to put in energy (or work) to stablize whole unit.

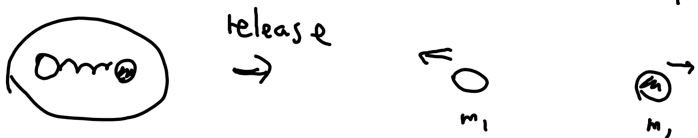
$\Rightarrow$ potential energy stored in the bound system

- as increase of rest mass : $M > m_1 + m_2$

(attractive case : need to take work out of it)

$\hookrightarrow M < m_1 + m_2$

total energy $= Mc^2 = K_1 + m_1 c^2 + K_2 + m_2 c^2 > (m_1 + m_2) c^2$



$$K_1 + K_2 = [\underbrace{M - (m_1 + m_2)}_{\Delta M \equiv M - (m_1 + m_2)}] c^2 \quad \text{energy release}$$

i.e. mass $\Delta M \Rightarrow$ converted to kinetic energy $K_1 + K_2$

(repulsive case) $\quad \underset{\underset{M-(m_1+m_2)}{\parallel\!\!|}}{\Delta M_R}\, c^2$ = work done to push bodies together

(attractive case) define $\Delta M_A \equiv m_1 + m_2 - M > 0$

$\Delta M_A c^2$ = binding energy B = work to pull bodies apart

= energy released as bodies come & form bound state

examples 2.6 – 2.8

2.11 w. massless particles

Example 2.6

The nuclei of certain atoms are naturally unstable, or radioactive, and spontaneously fly apart, tearing the whole atom into two pieces. For example, the atom called thorium 232 splits spontaneously into two "offspring" atoms, radium 228 and helium 4,

$$^{232}\mathrm{Th} \longrightarrow {}^{228}\mathrm{Ra} + {}^{4}\mathrm{He} \tag{2.32}$$

The combined kinetic energy of the two offspring is 4 MeV. By how much should the rest mass of the "parent" ^{232}Th differ from the combined rest mass of its offspring? Compare this with the difference in the measured masses listed in Appendix D.

Atom		Mass (in u)	
^{232}Th		232.038	Initial
^{228}Ra ^{4}He	228.031 + 4.003	232.034	Final Total
		0.004	Difference

This is the "repulsive" case

$\Delta M_K = M - (m_1 + m_2) = \frac{K_1 + K_2}{c^2} = 4\,\mathrm{MeV}/c^2$

$= 4 \times 1.778 \times 10^{-30}$ kg

$= \frac{4 \times 1.778 \times 10^{-30}}{1.66 \times 10^{-27}}\,u \approx \underline{4.28 \times 10^{-3}\,u}$

agrees with actual mass difference

1 u = 1.66×10^{-27} kg (atomic unit) defined as $\frac{1}{12}$ of Carbon 12 atom mass $= \frac{12\ g}{6.02 \times 10^{23}}$)

"attractive" case

Example 2.7

It is known that two oxygen atoms attract one another and can unite to form an O_2 molecule, with the release of energy $E_{out} \approx 5$ eV (in the form of light if the reaction takes place in isolation). By how much is the O_2 molecule lighter than two O atoms? If one formed 1 gram of O_2 in this way, what would be the total loss of rest mass and what is the total energy released? (The O_2 molecule has a mass of about 5.3×10^{-26} kg.)

$B = \Delta M_A c^2 = (m_1 + m_2 - M)c^2 \approx 5\,\mathrm{eV}$

$m_O = 15.994\,u = 2.655 \times 10^{-26}$ kg

$\Delta M_A = \frac{B}{c^2} = \frac{5\,\mathrm{eV}}{c^2} = 5 \times 1.778 \times 10^{-36}$ kg

$\simeq 8.89 \times 10^{-36}$ kg.

$M_{O_2} = 2m_O - \Delta M_A = 5.31 \times 10^{-26}$ kg $- 8.89 \times 10^{-36}$ kg

Fractional loss $\frac{8.89 \times 10^{-36}}{5.31 \times 10^{-26}} \sim 1.7 \times 10^{-10}$

1g → loses 1.7×10^{-10} gram only.

⇒ energy 1.7×10^{-10} g $= 1.7 \times 10^{-13}$ kg $= \Delta M$

$\Delta E = \Delta M c^2 = \frac{1.7 \times 10^{-13}}{1.778 \times 10^{-36}}$ eV $\sim 10^{23}$ eV $\sim 1.6 \times 10^{4}$ J

cf. 2260 J to boil 1g of water

Example 2.8

The Λ particle is a subatomic particle that (as mentioned in Example 1.2) can decay spontaneously into a proton and a negatively charged pion.

$$\Lambda \to \mathrm{p} + \pi^-$$

(This immediately tells us that the rest mass of the Λ is greater than the total rest mass of the proton and pion.) In a certain experiment the outgoing proton and pion were observed, both traveling in the same direction along the positive x axis with momenta

$$p_\mathrm{p} = 581\ \mathrm{MeV}/c \quad \text{and} \quad p_\pi = 256\ \mathrm{MeV}/c$$

Given that their rest masses are known to be

$$m_\mathrm{p} = 938\ \mathrm{MeV}/c^2 \quad \text{and} \quad m_\pi = 140\ \mathrm{MeV}/c^2$$

find the rest mass m_Λ of the Λ.

$\Lambda \to \quad \overset{p}{\longrightarrow} \quad \overset{\pi}{\longrightarrow}$

momentum

$$p_\Lambda = p_p + p_\pi = 581+256$$
$$= 837\ \mathrm{MeV}/c$$

energy

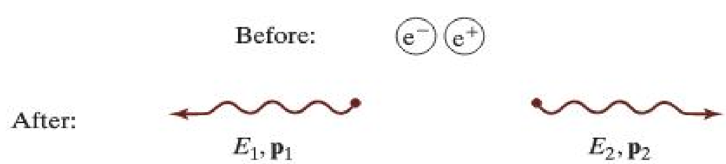
$$E = \sqrt{m_\Lambda^2 c^4 + p_\Lambda^2 c^2} = \sqrt{m_p^2 c^4 + p_p^2 c^2} + \sqrt{m_\pi^2 c^4 + p_\pi^2 c^2}$$

$$\sqrt{m_\Lambda^2 c^4 + 837^2} = \sqrt{581^2 + 256^2} + \sqrt{938^2 + 140^2} \approx 1395\ \mathrm{MeV}$$

$$\Rightarrow m_\Lambda \approx 1116\ \mathrm{MeV}/c^2$$

Example 2.11

As we will discuss in Chapters 17 and 18, there is a subatomic particle called the **positron**, or *antielectron*, with exactly the same mass as the electron (0.511 MeV/c^2) but the opposite charge. The most remarkable property of the positron is that when it collides with an electron, the two particles can annihilate one another, converting themselves into two or more photons. Consider the case that the electron and positron are both at rest and that just two photons are produced, with energies E_1 and E_2 and momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ (Fig. 2.6). Use conservation of energy and momentum to find the energies E_1 and E_2 of the two photons.



momentum $\quad p_1 + p_2 = 0 \quad \Rightarrow \quad p_1 = -p_2$

energy $\quad E_1 + E_2 = 2m_e c^2$

$$2 p_2 c = 2 m_e c^2 \quad \Rightarrow \quad |p_1| = |p_2| = m_e c = 0.511\ \mathrm{MeV}/c$$

$$E_1 = E_2 = 0.511\ \mathrm{MeV}$$

energy in the form of mass is converted to that in the form of electromagnetic waves!

Force again

$F = \frac{dp}{dt}$ earlier we used this to derive energy (via $dE = \vec{F}\cdot d\vec{r}$) then $E = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} = \sqrt{p^2c^2 + m^2c^4}$

Example 2.9 simply does the reverse: use energy expression E to show that $dE = \vec{F}\cdot d\vec{r}$

$$\frac{d}{dt}E = \frac{d}{dt}\sqrt{p^2c^2+m^2c^4} = \frac{1}{2}\frac{1}{\sqrt{p^2c^2+m^2c^4}}\, 2\,\vec{p}\cdot\frac{d\vec{p}}{dt}\,c^2 = \frac{c^2\vec{p}}{E}\cdot\frac{d\vec{p}}{dt} = \vec{u}\cdot\frac{d\vec{p}}{dt}$$

recall $\frac{c^2\vec{p}}{E} = \vec{u}$

So $\frac{dE}{dt} = \vec{u}\cdot\vec{F} = \frac{d\vec{r}}{dt}\cdot\vec{F} \Rightarrow dE = \vec{F}\cdot d\vec{r}$ ✗✗

Lorentz force $\vec{F} = q(\vec{\mathcal{E}} + \vec{u}\times\vec{B})$

e.g. with only $\vec{B}$, $\vec{F} = q\,\vec{u}\times\vec{B} = \frac{d\vec{p}}{dt} = m\frac{d}{dt}\left(\gamma(u)\,\vec{u}\right)$

Since $\vec{F}$ is perpendicular to $\vec{u}$, u's magnitude will not increase

[one way to see it is to recall classical circular motion, force only changes magnitude (no work done) $\Rightarrow$ speed remains the same;

Similarly here $\vec{F}\cdot\vec{u} = 0 \rightarrow \frac{dE}{dt} = 0 \quad E = \sqrt{p^2c^2+m^2c^4} = \text{constant}$

$\Rightarrow$ p's magnitude remains const (hence u)

Thus, $m\frac{d}{dt}(\gamma(u)\,\vec{u}) = m\gamma\frac{d\vec{u}}{dt} = q\,\vec{u}\times\vec{B}$

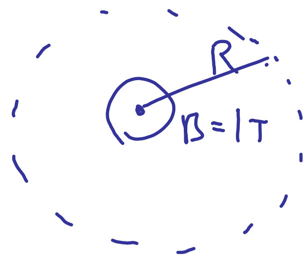
Assume particle moves in a plane $\perp$ to $\vec{B}$

$\Rightarrow m\gamma a = q u B \Rightarrow m\gamma\frac{u^2}{R} = q u B \Rightarrow \boxed{R = \frac{m\gamma u}{qB} = \frac{p}{qB}}$

or $p = qBR$ momentum can be measured for charge particles!

Example 2.10

proton momentum unknown (p)



$R = 1.4$

$m_p = 938.27 \ \mathrm{MeV}/c^2$

$$p = qBR = 1.6 \times 10^{-19} \times 1 \times 1.4 \ \mathrm{kg \cdot \frac{m}{s}}$$

$$= \frac{2.24 \times 10^{-19} \ \mathrm{kg \cdot m/s}}{5.34 \times 10^{-22} \ \mathrm{kg \cdot m/s} \,/\, \mathrm{MeV/c}} = 420 \ \mathrm{MeV/c}$$

$$E = \sqrt{(pc)^2 + m_p^2 c^4} = \sqrt{420^2 + 938.27^2} \ \mathrm{MeV} \cong 1030 \ \mathrm{MeV}$$

When is non-relativistic mechanics good enough?

It depends on the desired accuracy.

e.g. example. $v = 300 \ \mathrm{m/s}$ for an hour, time dilation is several nanoseconds $1\,\mathrm{ns} = 10^{-9}\,\mathrm{s}$

But for application in GPS, nanosecond error affects accuracy of location e.g. meter

TABLE 2.4

The relativistic and nonrelativistic kinetic energy of a mass m at various speeds u, in units of mc^2.

u:	0.01c	0.1c	0.5c
$K_{rel} = (\gamma - 1)mc^2$:	5.0004×10^{-5}	5.038×10^{-3}	0.155
$K_{NR} = \frac{1}{2}mu^2$:	5.0000×10^{-5}	5.000×10^{-3}	0.125
% difference:	0.01%	1%	20%

① if $v \ll 0.1c$ nonrelativistic mechanics ok.

② if $v \gtrsim 0.1c$ should use relativity

③ massless particles are always relativistic!

$\Leftrightarrow$ energy

① $K < 1\%$ of rest energy $mc^2 \Rightarrow$ non-relativistic

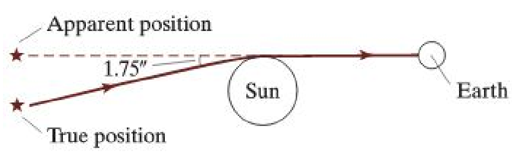
② $K \gtrsim 1\% \ mc^2 \rightarrow$ relativistic

A few words on general relativity

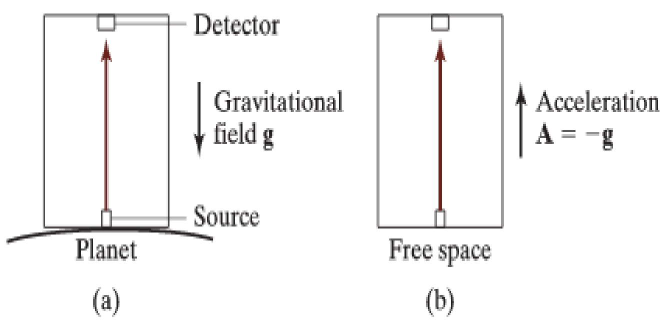
Presence of mass change curvature of spacetime and mass moves in the geodesic of spacetime. Mathematics involves tensor analysis and is more complicated. We won't need general relativity in PHY251.
But it certainly is one of the most important discoveries in 20th century.

Consequence of general relativity:

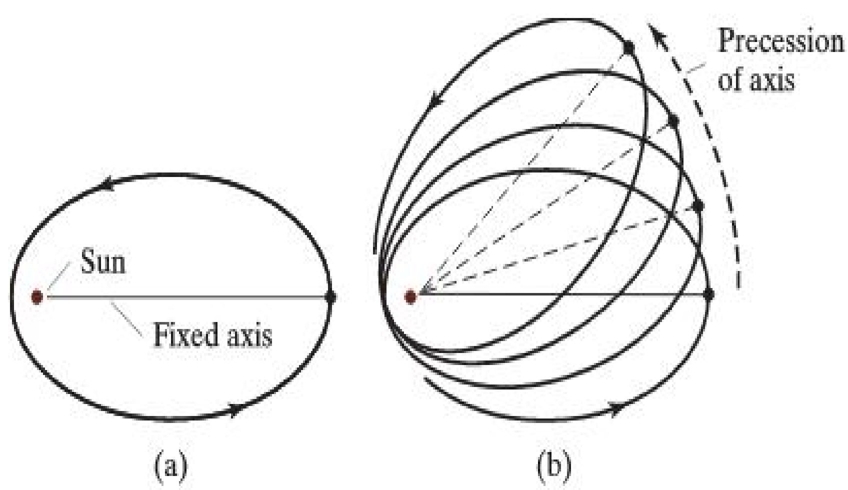
① Bending of light by gravity (nowadays important tool in astrophysics such as gravitational lensing in inferring dark matter)



② Gravitational Redshift

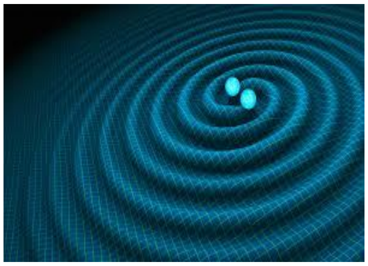


③ precession of Mercury's orbit



④ Black holes

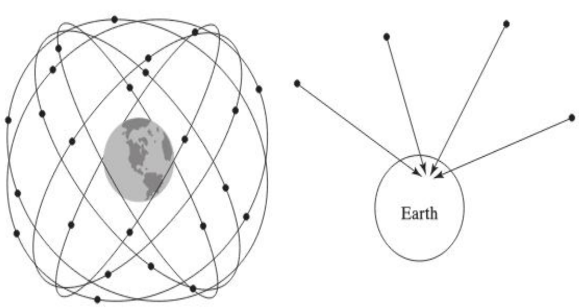
⑤ Gravitational wave (already detected using Michelson interferometer in LIGO !)



GP-B was designed to measure two key predictions of Einstein's general theory of relativity by monitoring the orientations of ultra-sensitive gyroscopes relative to a distant guide star. Learn more about the mission.

https://einstein.stanford.edu/

GPS is an application of special & general relativity



(a) (b)

See Probs 2.47 – 2.49 for further analysis.