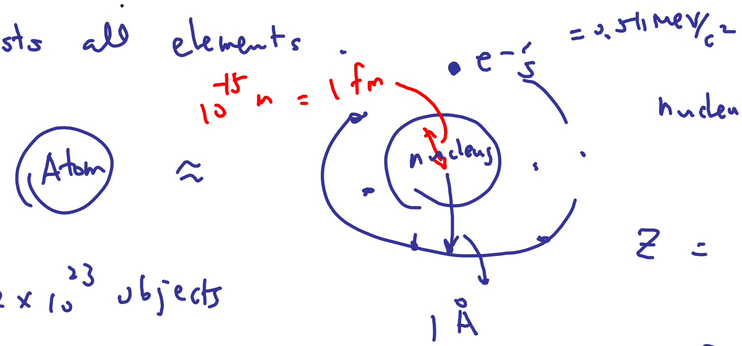
Electron, Rutherford's nuclear Atom, and quantization of light

We are all familiar with atoms and molecules, which are building blocks of everyday materials. Gold (Au), Silver (Ag), Copper (Cu), Aluminum (Al), Tin (Sn), etc. are metals. Air is composed of nitrogen molecules (N_2) and oxygen molecules. You probably still remember ideal gas law $PV = nRT = Nk_BT$. A water molecule is composed of two H and one O : H_2O & a macroscopic # of H_2O can exist in the form of ice (solid), water (fluid) and vapor (gas).

The Periodic Table lists all elements.



nucleus: protons $m_p = 938.3\,MeV/c^2$

neutron $m_n = 939.6\,MeV/c^2$

Z = atomic # = # of protons = # electrons

N = neutron #

A = mass # = $Z + N$

1 mole $\approx 6.022 \times 10^{23}$ objects

$1\,u = \frac{1}{12}$ (mass of ^{12}C atom)

binding energy

$$B = (Zm_e + Zm_p + Nm_n - M_{atom})c^2$$

We will discuss 3 topics from chapter 3 (Atoms):

3.10: Thomson's Discovery of electron

3.11: Millikan's Oil-Drop experiment

3.12: Rutherford & the nuclear atom

Thomson's Discovery of e^-

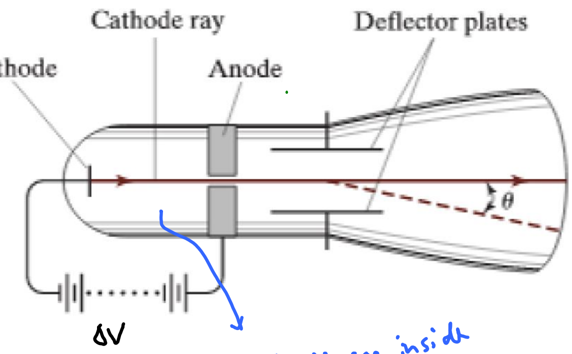


FIGURE 3.6 Thomson's cathode ray tube. When opposite charges were placed on the deflector plates, the "rays" were deflected through an angle θ as shown.

(1) When ΔV large is applied, some gas atoms ionize and electrons discharge

e^- accelerates and moves toward Anode and passes thru

(2) with deflector plates e^-'s are bent. [Bending of cathode rays shows that they are charged.]

But required a good vacuum otherwise e^-'s hit gas atoms ionizing them; ions attracted to a deflector plate and cancel deflecting electric field.

The ray can be used to charge a metal cup $e^- \to$

Also bent by a magnetic field. B.

(i) Using combinations of E & B, one can measure v of e^-'s

$v = \frac{E}{B}$ $\qquad \vec{F} = -e(\vec{E} + \vec{v} \times \vec{B})$

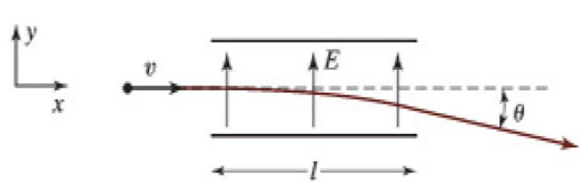
(a) with only $\vec{E}$

(b) turn off $\vec{E}$, apply B to get same θ

$\Rightarrow$ infer $v = \frac{E}{B}$

(ii) with v known, can measure radius of circular motion in field B $\Rightarrow$ $\underbrace{R}_{\text{measured}} = \frac{m}{e}\left(\frac{v}{B}\right)$ ← known $\Rightarrow$ infer m/e

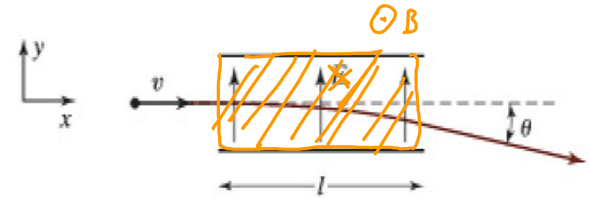
3.40 •• In Thomson's experiment electrons travel with velocity v in the x direction. They enter a uniform electric field E, which points in the y direction and has total width l (Fig. 3.15). Find the time for an electron to cross the field and the y component of its velocity when it leaves the field. Hence show that its velocity is deflected through an angle $\theta \approx eEl/(mv^2)$ (provided that θ is small). Assume that the electrons are nonrelativistic, as was the case for Thomson.



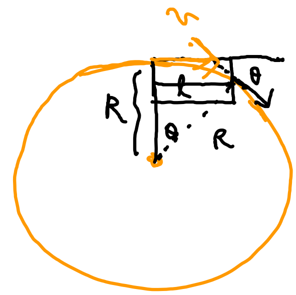
$v_x = v$, $\Delta t = \frac{l}{v}$

$v_y = \frac{eE}{m}\frac{l}{v}$, $\tan\theta = \frac{v_y}{v_x} = \frac{eEl}{mv^2}$

3.41 •• Suppose that the electrons in Thomson's experiment enter a uniform magnetic field B, which is in the z direction (with axes defined as in Fig. 3.15) and has total width l. Show that they are deflected through an angle $\theta \approx eBl/(mv)$ (provided that θ is small). Assume that the electrons are nonrelativistic.



$m\frac{v^2}{R} = evB$. $R = \frac{mv}{eB}$



$\sin\theta = \frac{l}{R} = \frac{leB}{mv}$

$\theta \simeq \frac{leB}{mv}$

Millikan's Oil-drop Experiment

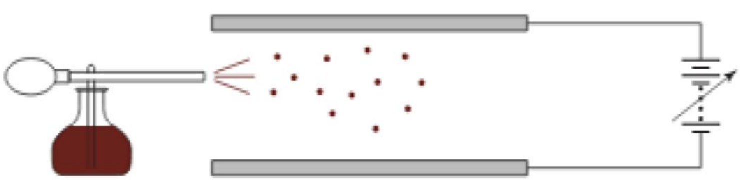


FIGURE 3.7
Millikan's oil-drop experiment. The potential difference between the plates is adjustable, both in magnitude and direction.

Thomson's experiment can only determine the ratio m/e, not m, e individually.
Millikan was able to measure e^-.

Oil drops will fall under gravity but due to friction/viscosity will reach a terminal velocity $v = \frac{Mg}{6\pi r\eta} = \frac{\frac{4}{3}\pi r^3 \rho g}{6\pi r\eta} = \frac{2r^2\rho g}{9\eta}$

ρ, g, η known

r inferred from measuring v

$\downarrow Mg$ M

But with e^- $\uparrow eE$ M e^- $\downarrow Mg$ and external field E to balance the force (adjust it)

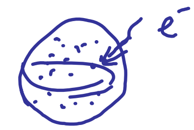
$qE = Mg \Rightarrow q = \frac{\frac{4}{3}\pi r^3 \rho g}{E}$ charge can be inferred $e = 1.6\times10^{-19}$ Coulomb

electron's charge $q_e = -e$

$\Rightarrow m_e = 9.11\times10^{-31}\,kg = 0.511\,MeV/c^2$

$\sim \frac{1}{2000}$ of hydrogen atom

Rutherford's experiment and the nuclear atom



Thomson has a model that e^- is distributed in uniform positive background (of sphere (of atom))

Rutherford and his assistants Geiger & Marsden carried out experiments to test this

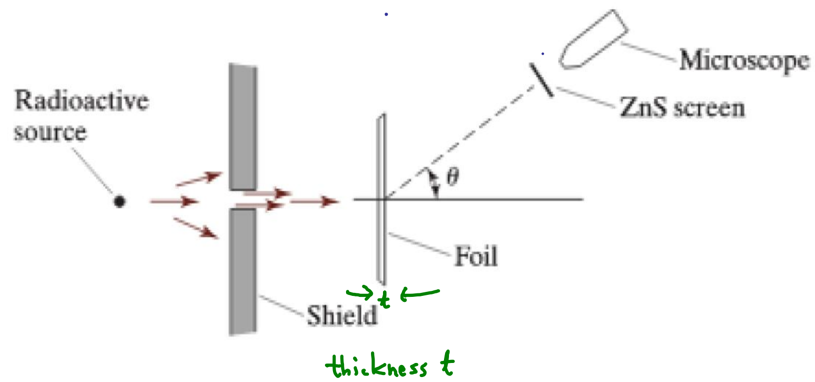
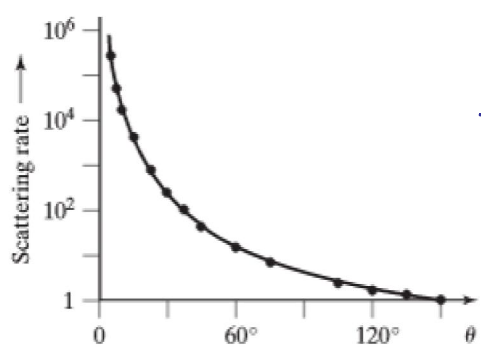


FIGURE 3.8
The "Rutherford scattering" experiment of Geiger and Marsden. Alpha particles from a radioactive source pass through a narrow opening in a thick metal shield and impinge on the thin foil. The number scattered through the angle θ is counted by observing the scintillations they cause on the zinc sulfide screen.

FIGURE 3.11
Geiger and Marsden measured the flux of scattered alpha particles at 14 different angles. Their measurements fit Rutherford's predicted $1/\sin^4(\theta/2)$ behavior beautifully. Notice the vertical scale is logarithmic and the measurements span an amazing 5 orders of magnitude.

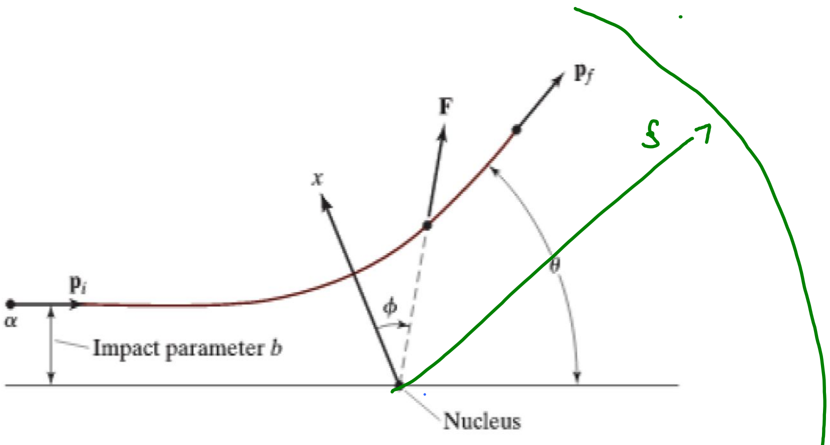


- used α ($^4He^{2+}$) particle to bombard on thin metal foil
- most deflected angles θ are small, but some are as large as 90° & even 180° (even thinnest foil)
- not consistent with uniform positive charge.

$\Rightarrow$ Rutherford concluded with a model of atom:

e^- ⊕ nucleus carries positive charge in a small volume & e^-'s outside the positive nucleus.

Can this lead to some quantitative prediction to compare with experiments?



α particle can probe the nucleus closely, & see bare Ze charge.

Rutherford derived a formula for # of particles per unit area at θ i.e. $n_{sc}(\theta)$: (Rutherford formula)

$$n_{sc}(\theta) = \text{number of particles per unit area at } \theta$$
$$= \frac{Nnt}{4s^2} \cdot \left(\frac{Zke^2}{E}\right)^2 \cdot \frac{1}{\sin^4(\theta/2)} \qquad (3.50)$$

$b = \dfrac{Zke^2}{E \tan \theta/2}$ b: impact parameter

Derived using

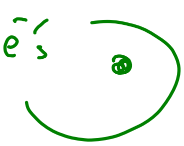
- Coulomb repulsion $F = \dfrac{kqQ}{r^2} = \dfrac{2Zke^2}{r}$

α-particle has charge $\underline{2e}$

$Ze \to$ protons

- target foil has thickness t & contains n atom in unit volume.
- s: distance from nucleus to detector , E: energy of α particle.

[For derivation, see 3.12] The formula agrees w. data!

But with e^-'s  orbiting nucleus is not stable according to

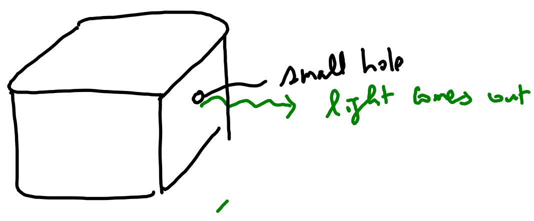
$\underline{E\&M}$.

II light & its quantization

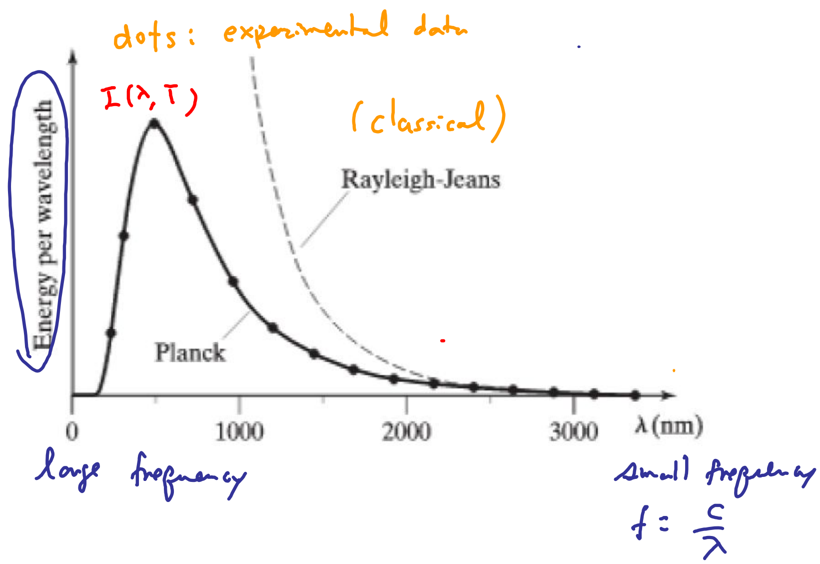
- Planck & Blackbody Radiation
- The Photoelectric Effect
- X-rays & Bragg Diffraction
- X-ray Spectra
- The Compton Effect
- Particle-wave Duality

Quantization of light

According to classical theory of radiation, the energy of light is continuous. However experiments around 1900 indicated failure of such theory to explain the so-called black-body radiation



which was one of many catastrophes of classical mechanics!

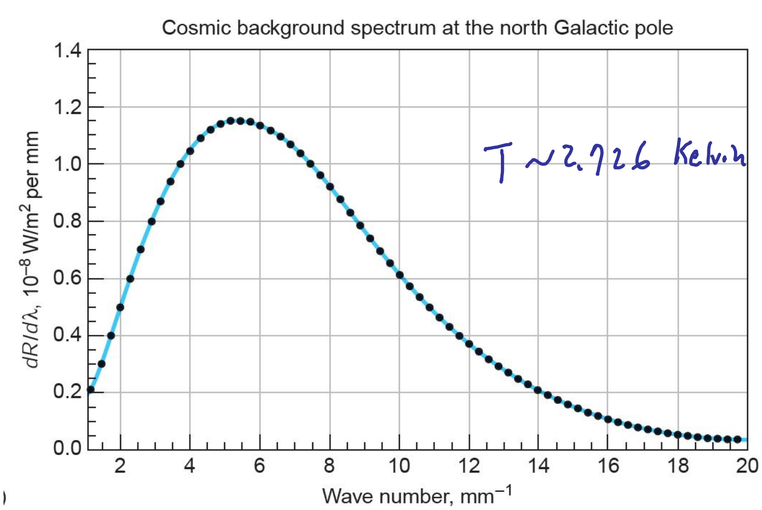


Max Planck was able to explain the observed distribution by invoking the notion that light with frequency f has energy in chunks:

$$E = 0,\ hf,\ 2hf,\ 3hf$$

(with h called Planck constant)

$$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s}$$



$$I(\lambda, T) = \frac{2\pi h c^2}{\lambda^5}\ \frac{1}{e^{hc/\lambda k_B T} - 1}.$$

power radiated per unit area with wavelength btwn λ & $\lambda + d\lambda$

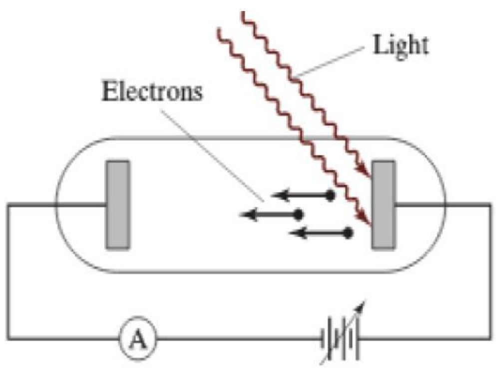
Examples of black-body radiation include our sun and the so-called cosmic microwave background (CMB); the latter provides the evidence that our universe experienced a period of fast expansion. (inflation)?

[We will be able to derive this when we learn quantum statistical mechanics.]

$\Rightarrow$ Energy of light is quantized $\Rightarrow$ notion of photons

Photoelectric effect

Einstein took up Planck's idea and used it to explained the photoelectric effect. (by Heinrich Hertz in 1887)



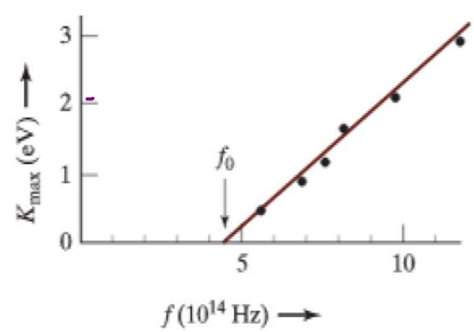


FIGURE 4.3
Millikan's data for K_{max} as a function of frequency f for the photoelectric effect in sodium.

- metal exposed to light can eject electrons (detected by a current $\rightarrow$ # of e^- if properly biasing 2nd electrode)
- 2nd electrode kept at lower potential $\Rightarrow$ repel e^- at a stopping potential V_s : current ceases

$V_s e = K_{max}$ (the potential energy balances the max kinetic energy)

* 1. If intensity of light is increased. # of e^- increases, but their kinetic energy does not change!

* 2. If frequency f is below a threshold f_0 $\Rightarrow$ no e^- ejected no matter how high intensity

These cannot be explained by a continuous energy of light (i.e. proportional to intensity). But quantization of light energy $E = hf$ can easily explains 1 & 2.

$$h f_0 = \phi \leftarrow \text{work function}$$, i.e. minimum energy needed to overcome potential energy of e^- inside metal.

$$hf - \phi = K_{max}$$

Example 4.1 typical visible photon? how many photons (per second) entering eyes in light intensity 3×10^{-4} watt $/m^2$?

e.g. 400 ⟶ 700 nm

$\lambda \sim 550\,nm$, $E = hf = h\frac{c}{\lambda}$

$hc = 6.63 \times 10^{-34}\,J.s. \; 3 \times 10^{8}\,m/s$

$= 1.989 \times 10^{-25}\,J \cdot m$

$= 1.24 \times 10^{-6}\,eV \cdot \underline{m}$

$= 1240\,eV\,\underline{nm}$

$$E = \frac{1240\,eV}{550} \cong 2.3\,eV.$$

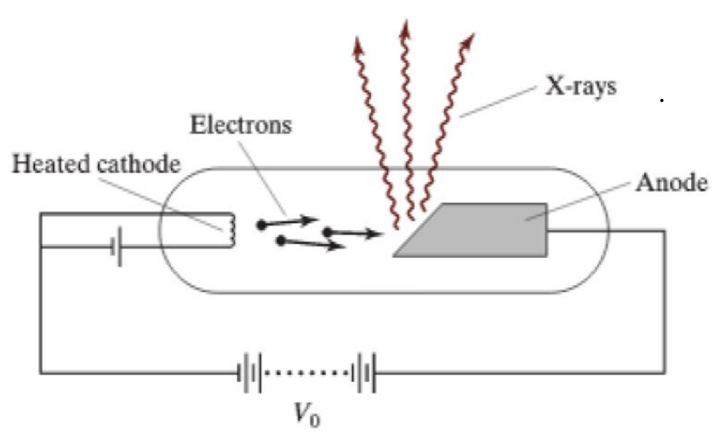
$$\frac{E}{t} = I \cdot A = 3 \times 10^{-4} \frac{watt}{m^2} \cdot 3 \times 10^{-5}\,m^2 \sim 9 \times 10^{-9}\,watt$$

e.g. 6 mm diameter, # per second $\sim \frac{9 \times 10^{-9}\,J}{1.989 \times 10^{-25}\,J \cdot m} \sim 2.5 \times 10^{10}$

Example 4.2 $\phi = 4.7\,eV$ silver $V_s = ?$ for $\lambda = 200\,nm$

$$V_s = \frac{hf - \phi}{e} = \frac{hc}{\lambda e} - \frac{\phi}{e} = \frac{1240\,eV.nm}{200\,nm} - 4.7\,V = (6.2 - 4.7)\,V = 1.5\,V$$

4.4 X-rays and Bragg Diffraction



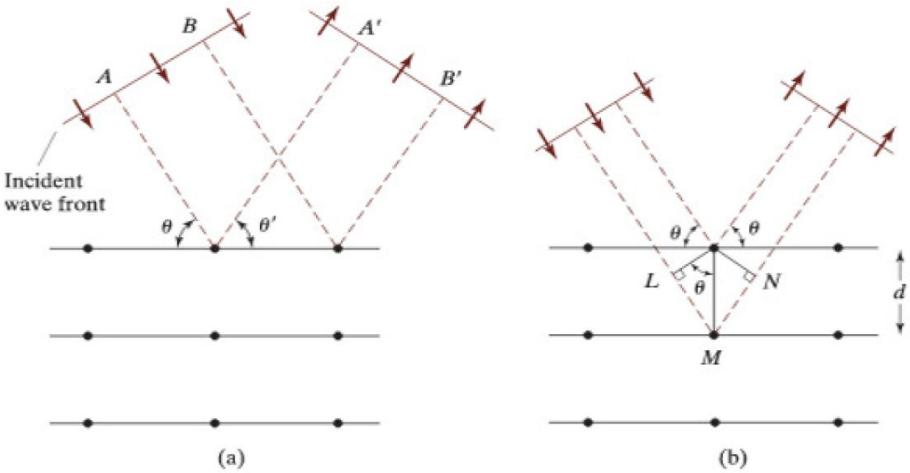
X-rays are light with wavelength $0.001\,nm \sim 1\,nm$ ⟶ high energy

1240 eV to 1240000 eV !

generated by high speed electrons colliding onto metal [Bremsstrahlung = braking radiation]

Due to small wavelength (can be comparable to spacing between atoms in crystal), X-rays are good for probing crystal structure, using interference, specifically called Bragg diffraction.

Bragg diffraction



peaks located at θ's:

$$2d \sin\theta = n\lambda$$

(Bragg law)

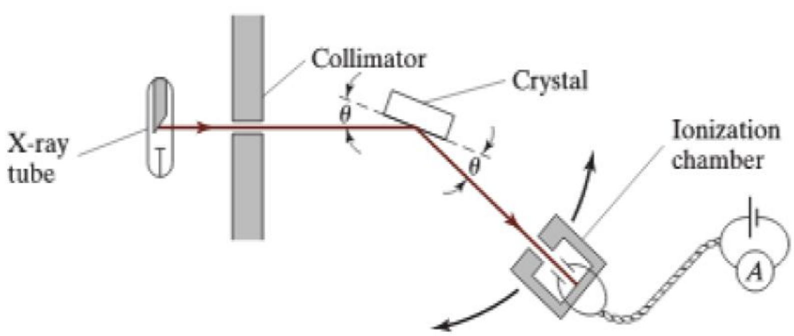
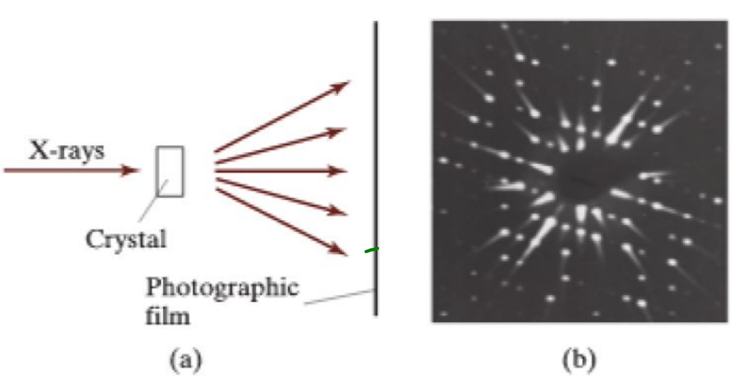
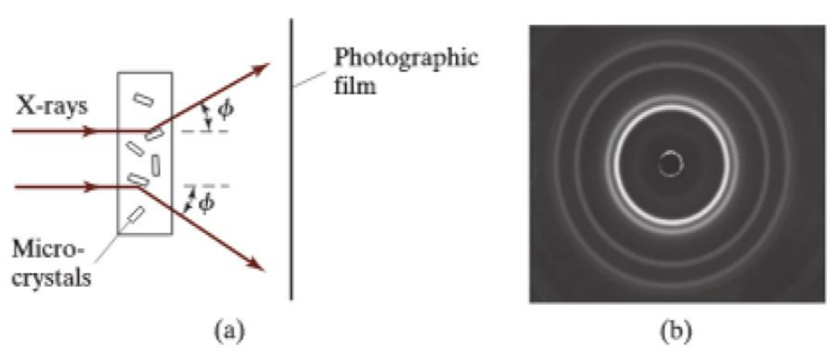


FIGURE 4.7
An X-ray spectrometer. X-rays are reflected off the crystal and detected by the ionization chamber. The crystal and chamber can both rotate in such a way that the two angles θ are always equal.

Crystal structure, DNA structure ...





X-ray spectra

$h f_{max} = K = V_0 e$ Duane-Hunt law

4.5 X-ray Spectra

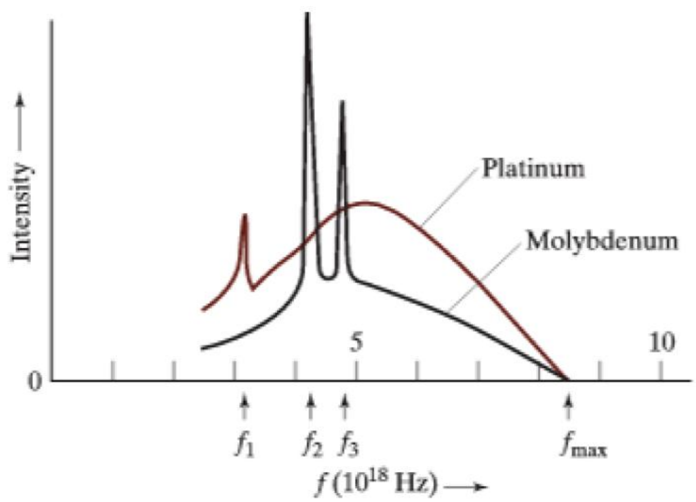
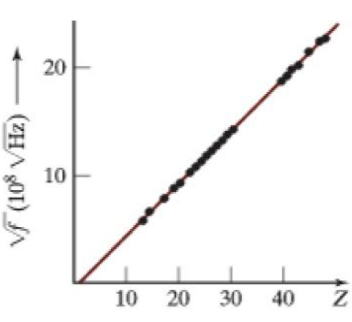


FIGURE 4.11
X-ray spectra produced by platinum and molybdenum anodes, both made with an accelerating potential of 35 kV. Note how both spectra terminate abruptly at the same frequency, f_{max}.

Classical theory of bremsstrahlung predicts a broad spread of frequencies with smooth intensity.

But observed: ① sharp peaks

② exists a threshold f_{max} $\quad hf_{max} = K = V_0 e$



$$\sqrt{f} \propto (Z - 1),$$

FIGURE 5.7
Moseley measured the frequencies f of K_α X-rays, using several different elements for the anode of his X-ray tube. The graph shows clearly that $\sqrt{f}$ is a linear function of the atomic number Z of the anode material. The reason the line crosses the Z axis at $Z \approx 1$ is explained in the text.

$$E_\gamma = \frac{3}{4}(Z - \delta)^2 E_R$$

Transitions between $n = 2$ and $n = 1$ are traditionally identified as the K_α

$$E_\gamma = E_2 - E_1 = Z^2 E_R\left(1 - \frac{1}{4}\right) = \frac{3}{4} Z^2 E_R \qquad E_n = -Z^2 \frac{E_R}{n^2}$$

frequencies of their K_α X-rays (or any other definite X-ray line), then the photon energies, and hence frequencies, should vary like the square of the atomic number Z; that is, we should find $f \propto Z^2$, or equivalently

$$\sqrt{f} \propto Z \qquad (5.35)$$

4.6 The Compton Effect

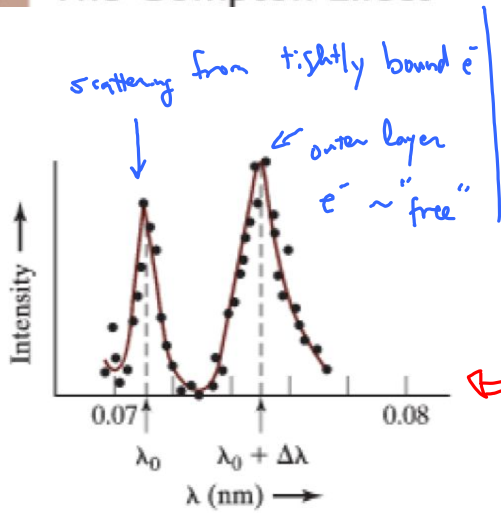


FIGURE 4.14

The spectrum of X-rays scattered at $\theta = 135°$ off graphite, as measured by Compton; λ_0 is the incident wavelength (0.0711 nm) and $\Delta\lambda$ is the shift predicted by Compton's formula (4.25).

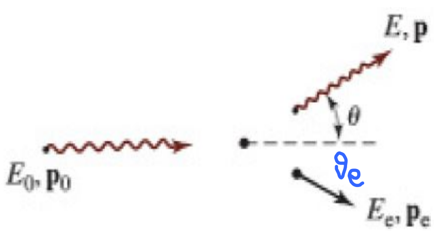


FIGURE 4.12

Compton scattering, in which a photon scatters off a free electron.

Classical theory predicts that when a light (with freq f_0) is fired at a system of charges, the frequency of the scattered light $\boxed{f = f_0}$ (classical)

But this is not what was observed. A range of frequency (or equivalent wave) was seen. (careful measurement)

Moreover it was also seen that $f < f_0$

Compton was able to explain this using quantization of light. For light $(m = 0)$

$$E^2 = (pc)^2 + (mc^2)^2 = (pc)^2$$

$$E = pc = hf \Rightarrow p = \frac{hf}{c} = \frac{h}{\lambda}$$

Assume photon collides w. e⁻ just like two particle collision, momentum and energy are conserved.

momentum

$$x: \quad p_0 = \frac{h}{\lambda_0} = p\cos\theta + p_e\cos\theta_e$$

$$y: \quad 0 = p\sin\theta - p_e\sin\theta_e$$

(or $\vec{p}_0 = \vec{p}_e + \vec{p}$)

energy:

$$p_0 c + mc^2 = pc + \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

From momentum: $(\vec{p}_e)^2 = (\vec{p}_0 - \vec{p})^2 = p_0^2 + p^2 - 2\vec{p}_0\cdot\vec{p} = p_0^2 + p^2 - 2p_0 p\cos\theta$

or $\begin{cases} p_e\cos\theta_e = p_0 - p\cos\theta \\ p_e\sin\theta_e = p\sin\theta \end{cases} \Rightarrow p_e^2 = (p_0 - p\cos\theta)^2 + (p\sin\theta)^2$

From energy $\left[\sqrt{p_e^2c^2+m_e^2c^4}\right]^2 = \left[(p_0-p)c + m_ec^2\right]^2$

$$p_e^2c^2+m_e^2c^4 = (p_0-p)^2c^2 + 2(p_0-p)m_ec^3 + m_e^2c^4$$

$$p_e^2 + \cancel{m_e^2c^2} = (\cancel{p_0^2} - 2p_0p + \cancel{p^2}) + \underline{2(p_0-p)m_ec} + \cancel{m_e^2c^2}$$

$''$

$\cancel{p_0^2}+\cancel{p^2}-2p_0p\cos\theta$

$$\Rightarrow \quad m_ec(p_0-p) = p_0p(1-\cos\theta)$$

$$\Rightarrow \quad \frac{1}{p} - \frac{1}{p_0} = \frac{1}{m_ec}(1-\cos\theta)$$

$$\Rightarrow \quad \frac{\lambda}{h} - \frac{\lambda_0}{h} = \frac{1}{m_ec}(1-\cos\theta)$$

$$\Rightarrow \quad \Delta\lambda \equiv \lambda-\lambda_0 = \frac{h}{m_ec}(1-\cos\theta) \geq 0$$

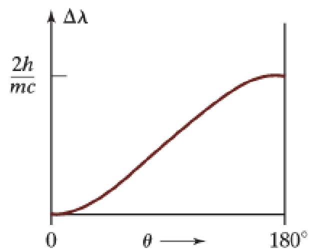


FIGURE 4.13
Increase, $\Delta\lambda$, in the wavelength of photons in Compton scattering. Note that $\Delta\lambda$ is zero at $\theta = 0$ and rises to a maximum of $2h/mc$ at $\theta = 180°$.

Since $\lambda \geq \lambda_0 \Rightarrow f \leq f_0$!

magnitude $\frac{h}{m_ec} = \frac{hc}{m_ec^2} = \frac{1240 \text{ eV}\cdot\text{nm}}{0.511 \text{ MeV}} = 0.00243 \text{ nm}$

[difficult to observe for visible light very small.
For X-rays it's possible]

Wave-particle duality

Light, usually regarded as a wave, displays particle-like properties such as in Compton effect & photoelectric effect.

Light properties λ & f $\Rightarrow$ particle-like $p = \frac{h}{\lambda}$ & $E = hf$

* In fact de Broglie reversed this:
particles also exhibit wave properties (6.2 De Broglie's hypothesis)

$p, E \Rightarrow \lambda = \frac{h}{p}$, $f = \frac{E}{h}$

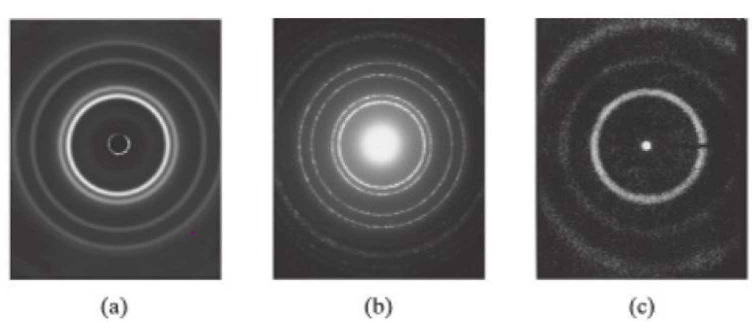


FIGURE 6.3
Diffraction rings produced by diffraction of waves in polycrystalline metal samples with **(a)** X-rays, **(b)** electrons, **(c)** neutrons.