Limits on the Rest Mass of the Photon: from
to Vulcan-Solar Probe

Alfred S. Goldhaber and MMN
PRL 21, 567 (1968)
26, 1390 (1971)
RMP 43, 227 (1971)
PRD 9, 1119 (1974)
with L. Davis, Jr. PRL 35, 1402 (1975)

1) Maxwell Eqs. → Proca Eq. 

\( \mu \neq 0 \)

2) Consequences of \( \mu \neq 0 \)

3) Limits on \( \mu \) from
   a) \( c = \text{Const.} \)
   b) Coulomb's Law
   c) Earth's magnetic field
   d) Jupiter's magnetic field
THIS CAN ALL BE DONE
RIGOROUSLY (See RMP)

Special relativity

Fields Linear in Currents $\Rightarrow$ PROCA

Energy Conservation
PROCA Eq. [Maxwell $\mu \neq 0$]

$$\partial^\gamma F_{\gamma \nu} + \mu^2 A_\nu = \left( \frac{4\pi}{c} \right) J_\nu$$

$$F_{\gamma \nu} = \partial_\gamma A_\nu - \partial_\nu A_\gamma$$

$$\Box + \mu^2 A_\nu = \frac{4\pi}{c} J_\nu \quad \text{[Charge conserved]}$$

$$\partial^\gamma A_\gamma = 0 \quad \text{[Lorentz gauge]}$$

$$\text{new "Maxwell's Eq"}$$

$$\begin{align*}
\nabla \cdot E &= 4\pi \rho - \mu^2 V \\
\nabla \times E &= -\frac{i}{c} \frac{\partial H}{\partial t} \\
\n\nabla \cdot H &= 0 \\
\n\nabla \times H &= \frac{i}{c} \frac{\partial E}{\partial t} + \frac{4\pi}{c} J - \mu^2 A
\end{align*}$$

STILL

$$\begin{align*}
H &= \nabla \times A \\
E &= -\nabla V - \frac{i}{c} \frac{\partial A}{\partial t}
\end{align*}$$
(\nabla^2 - \mu^2) A_\lambda = -4\pi J_\lambda / c

V = \frac{\exp(-\mu n)}{n}

\underline{YUKAWA POTENTIAL}

F = -\nabla V

= \frac{e^{-\mu n}}{n^2} \left[ 1 + \mu n \right]

\approx \frac{1}{n^2} \left[ 1 + \frac{\mu^2}{2} \right] + ... \n
⇒ NEED HIGH PREC. OR BIG APPARATUS

\Rightarrow 3rd Polarization ---
SIDELIGHT

M

1. Big Apparatus
   (ASC/WM)
   ~ Clusters
   CDM?

2. Small Apparatus
   + High Precision

\langle \text{Merc} \rangle = D. Croom
XV. Experiments with an Electrified Cup.

I shall close the account of my experiments with a small set, in which, as well as in the last, I have little to boast besides the honour of following the instructions of Dr. Franklin. He informed me, that he had found cork balls to be wholly unaffected by the electricity of a metal cup, within which they were held; and he desired me to repeat and ascertain the fact, giving me leave to make it public.

May we not infer from this experiment, that the attraction of electricity is subject to the same laws with that of gravitation, and is therefore according to the squares of the distances; since it is easily demonstrated, that were the earth in the form of a shell, a body in the inside of it would not be attracted to one side more than another.
II) Franklin. (empf. 1694 ±)

Franz Aepinus (1759: Latin in St. Petersburg) speculated $\alpha^{-2}$

John Robison (classicist 1769)
published in 1803 (Eng. Brit. mps.)

$$F \propto \frac{e_1 e_2}{L^2 + 1} \quad q = 0.06$$
III) CAVENDISH (1773)

Published by Maxwell in 1879

\[ |q| < \frac{1}{50} = 0.02 \]

1. Shells closed with contact. Charge placed on outer sphere.

2. Break connection.

3. Open outer sphere + see if any charge on inner sphere.
IV) **Coulomb** (1785- pub. 1789)

\[ q = 0.1 \]

Torsion balance at various distances --- Both repulsion + attraction.
VI  Plimpton and Lowton (1936)

\[ q = F(x, \theta) \frac{\Delta V}{V} \]
\[ = 2 \times 10^{-9} \]

\[ b = 2 \pi r \]
\[ a = 2 \frac{1}{2} \pi r \]
\[ \Delta V = 10^{-6} V \]
Now, in our language

$$(\nabla^2 - \mu^2) V_n = 0$$

$$\phi(n) = K \left[ \frac{e^{\mu n} - e^{-\mu n}}{2 \mu n} \right]$$

$\alpha \leq n \leq \beta$

BECAUSE $V = C \text{const}$

$$V = \phi(a) \quad \Delta V = \phi(a) - \phi(b)$$

$$\frac{\Delta V}{V} = \frac{1}{6} \mu^2 (a^2 - \beta^2) + O[(\mu a)^4]$$

$$\mu \leq 10^{-6} \text{ m}^{-1} = 2 \times 10^{-11} \text{amu} \quad \Delta V = 4 \times 10^{-27} \text{m}$$
In 1960's IMPROVE

1. Lock in detector
2. \( w > 0 \), noise down
3. 1, 2, 3 order better

\[ M \leq 2 \times 10^{-17} \, g \]
\[ = 5 \times 10^{-16} \, m^{-1} \]
\[ = 9 \times 10^{-15} \, eV \]

**BEST LAB LIMIT**
CONSEQUENCES, $\mu \neq 0$

a) $N_\gamma \neq c$

$$(\Box - \mu^2) A_\lambda = 0$$

$$\left(\frac{\omega}{c}\right)^2 - \frac{k^2}{\omega^2} = \mu^2$$

$$N_\gamma = \frac{d\omega}{d\lambda} = \frac{c^2 k}{\omega} = c \frac{k}{(k^2 + \mu^2)^{1/2}}$$

DISPERSION OF LIGHT

PHOTONS ARE REL. PARTICLES
(3) a) dispersion of light signals

\[ s \lambda = \frac{d}{d \lambda} \left[ \frac{1}{N_b} - \frac{1}{N_R} \right] \approx \frac{1}{c^2} (N_b - N_R) \]

\[ = \frac{\mu^2 L}{8 \pi^2 c} \left( \lambda_b^2 - \lambda_R^2 \right) \left( \frac{c}{\lambda} \right)^2 \]

De Broglie (1940)

\[ \lambda = \frac{4000 \text{ Å}}{8000 \text{ Å}} \]

\[ L = 10^3 \text{ cm}\text{ yr} \]

\[ \mu < 0.78 \times 10^{-39} \text{ gm} \]

\[ \neq 10^{-44} \text{ gm} \]

DOUBLE STAR

\[ s \lambda < 10^{-3} \text{ Å} \]
Problem in plasma dispersion:

\[ A^2 = \frac{w_p^2}{c^2} \left[ 1 - \frac{w_p^2}{w^2 \pm w w_B} \right] \]

\[ w_p^2 = \frac{4\pi m e^2}{\hbar} \quad w_B = \frac{e B}{m c} \cos \theta \]

\[ n(t) = \frac{dw}{dA} = c \sqrt{1 - \frac{w_p^2}{w^2}} = c \left[ 1 - \frac{1}{2} \frac{w_p^2}{w^2} \right] \]

Same as Eq. for \( n_g \)

\[
\left( \frac{m c^2}{\hbar} \right) = 8.2 \times 10^{10} [\text{g}(\text{cm})] \text{sec}^{-1}
\]

\[
\left( \frac{4\pi m e^2}{\hbar} \right)^{1/2} = 5.6 \times 10^4 [m(\text{cm}^{-1})]^{1/2} \text{sec}^{-1}
\]

\[
\begin{bmatrix}
\left( m_c^2 \right)
\left( \frac{4\pi m e^2}{\hbar} \right)^{1/2}
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
0.028 \text{ electron/cm}^2
5.6 \times 10^4 [m(\text{cm}^{-1})]^{1/2}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mu = 10^{-44} \text{ g m}^{-1} \\
\phi = 3 \times 10^{-12} \text{ cm}^{-1} \\
\phi = 6 \times 10^{-12} \text{ cm}^{-1}
\end{bmatrix}
\]

CRAB PULSAR - NRD532
3-c) Earth's Magnetic Field

Schrödinger wanted a massive photon for his unified theory. (See Moore)

In 1943, inspired by McConnell's idea,

1. \[ \mu \neq 0 \Rightarrow \bar{A} = \bar{D} \times \bar{B}(\bar{E} \cdot \bar{n}) \]

2. \[ \vec{D} \cdot \bar{n} = 0 \]
   SO FLUX CONSERVED

3. So, lines closer in mostly at Equator
\[ D \Xi = D = -\frac{1}{2} \int d^3x \, \vec{J} \times \vec{A} \left( \frac{4\pi}{c} \right) \]

\[ (-\nabla^2 + \mu^2) \, A = J \left( \frac{4\pi}{c} \right) \]

\[ \Rightarrow A = \nabla \times D \left[ e^{-\mu r}/r \right] \]

**BUT** \[ H = \nabla \times A \]

\[ H = \left[ De^{-\mu r}/r^3 \right] \left[ \left( 1 + \mu r + \frac{1}{3} \mu^2 r^2 \right) \left( 3 \hat{n} \cdot \hat{n} - 2 \right) \right] \]

\[ -\frac{2}{3} \mu^2 r^2 \hat{n} \]

\[ H_{D} = \left[ D / r^3 \right] \left[ 3 \hat{n} \cdot \hat{n} - 2 \right] \]

**DIFFERENCES →** \[ H_{\text{EXTERNAL}} \left( \frac{m \text{ sphere}}{\text{anti-1 to D}} \right) \text{ at equator axis} \]

\[ \frac{H_{\text{M.T.}}}{H_{D.E.}} = \frac{\frac{2}{3} \mu^2 R^2}{1 + \mu R + \frac{1}{3} (\mu R)^2} \]

[Diagram of a sphere with arrows indicating field lines.]
I) Schrödinger had Ad Schmidts' (1885, 1922)
\[
\frac{\text{Hart}}{\text{Hoe}} \approx \frac{537 \, \text{g}}{310 \, \text{g}}
\]

\[(E_0 \text{ above}) + (\text{Path of 2}) \Rightarrow \mu \approx 10^{-9} \text{ gm}
\]

II) ASC + MMN had NASA's data (Can et al).

\[\text{Fit to } \sim 100 \text{ spherical harmonics.}
\]

ERA (1960.0)

\[
D = 31044 \, \text{g} \times R^2
\]

\[
\text{Hart} \cdot D = (21 \pm 5) \, \text{g}
\]

\[
\text{Hart} \cdot \left[ \frac{3 \times 8}{15 \times 8} \right] = \text{Hart} \cdot \hat{z} = (14 \pm 5) \, \text{g}
\]

\[
\text{Hart} \cdot \hat{z} \cdot D = (8 \pm 5) \, \text{g}
\]
3-d) Jupiter's Magnetic Field


Fit to $\mu$, internal & external dipoles and quadrupoles, taking out ring currents, etc.
VERY conservative fit obtained

\[ \mu \leq 8 \times 10^{-49} \text{ g} \]

\[ = 6 \times 10^{-16} \text{ eV} \]

\[ = 2 \times 10^{-11} \text{ cm}^{-1} \]
Fig 3.3 Trajectory geometry for in-ecliptic SPM with perihelion of 4 Solar radii.
What don't we understand about gravity?

Actually, we don't understand gravity and matter (let alone antimatter) for almost all of the universe!!

DMP
By the way, the biggest systematic in our acceleration residuals is a bias of

\[ 8 \times 10^{-13} \text{ km/s}^2 \]

directed toward the Sun.

\[ a_N = 5.93 \times 10^{-6} \frac{\text{m}}{\text{s}^2} \]
UNMODELED ACCELERATIONS ON PIONEER 10 AND 11

Acceleration Directed Toward the Sun

Acceleration (1.0E-13 km/sec/sec)

Heliocentric Distance (AU)

- Pioneer 10
- Pioneer 11
John D. Anderson (JPL)
Philip A. Laing (Aerospace)®
Eunice L. Lau (JPL)
Anthony S. Liu (Astrody. Sci.)®
MN
Slava G. Turyshev (JPL)
AFTER 2 CODES (98 POL) [87-94]

PIONEER 10 (ODP) [87-90]

\[(8.09 \pm 0.20) \times 10^{-8} \text{ cm/sec}^2\]

PIONEER 11 (ODP) [87-90]

\[(8.56 \pm 0.15) \times 10^{-8} \text{ cm/sec}^2\]

PIONEER 10 (CHASMP)

\[8.65 \pm 0.03\]

NOW NEED SYSTEMATICS P10 10 DATA \rightarrow 1998
still returning
good Doppler

receiver failure
no more Doppler
Fig. 1. The Pioneer 10 spacecraft.
The spin history of Pioneer 10.
<table>
<thead>
<tr>
<th>Program/Estimation method</th>
<th>Pio 10 (I)</th>
<th>Pio 10 (II)</th>
<th>Pio 10 (III)</th>
<th>Pio 11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sigma, WLS, no solar corona model</td>
<td>8.02 ± 0.01</td>
<td>8.65 ± 0.01</td>
<td>7.83 ± 0.01</td>
<td>8.46 ± 0.04</td>
</tr>
<tr>
<td>Sigma, WLS, with solar corona model</td>
<td>8.00 ± 0.01</td>
<td>8.66 ± 0.01</td>
<td>7.84 ± 0.01</td>
<td>8.44 ± 0.04</td>
</tr>
<tr>
<td>Sigma, BSF, 1-day batch, with solar corona model</td>
<td>7.82 ± 0.29</td>
<td>8.16 ± 0.40</td>
<td>7.59 ± 0.22</td>
<td>8.49 ± 0.33</td>
</tr>
<tr>
<td>CHASMP, WLS, no solar corona model</td>
<td>8.25 ± 0.02</td>
<td>8.86 ± 0.02</td>
<td>7.85 ± 0.01</td>
<td>8.71 ± 0.03</td>
</tr>
<tr>
<td>CHASMP, WLS, with solar corona model</td>
<td>8.22 ± 0.02</td>
<td>8.89 ± 0.02</td>
<td>7.92 ± 0.01</td>
<td>8.69 ± 0.03</td>
</tr>
<tr>
<td>CHASMP, WLS, with corona, weighting, and F10.7</td>
<td>8.25 ± 0.03</td>
<td>8.90 ± 0.03</td>
<td>7.91 ± 0.01</td>
<td>8.91 ± 0.04</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\mathcal{J}_{rc}^{1/c} &= (7.84 \pm 0.01) \times 10^{-8} \text{ cm/s}^2 \\
\alpha_{rc(157)} &= 5.65
\end{align*}
\]
Table 2: Error Budget: A Summary of Biases and Uncertainties.

<table>
<thead>
<tr>
<th>Item</th>
<th>Description of error budget constituents</th>
<th>Bias $10^{-8}$ cm/s$^2$</th>
<th>Uncertainty $10^{-8}$ cm/s$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Systematics generated external to the spacecraft:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) Solar radiation pressure and mass</td>
<td>+0.03</td>
<td>±0.01</td>
</tr>
<tr>
<td></td>
<td>b) Solar wind</td>
<td></td>
<td>± &lt; 10$^{-5}$</td>
</tr>
<tr>
<td></td>
<td>c) Solar corona</td>
<td></td>
<td>±0.02</td>
</tr>
<tr>
<td></td>
<td>d) Electro-magnetic Lorentz forces</td>
<td></td>
<td>± &lt; 10$^{-4}$</td>
</tr>
<tr>
<td></td>
<td>e) Influence of the Kuiper belt's gravity</td>
<td></td>
<td>±0.03</td>
</tr>
<tr>
<td></td>
<td>f) Influence of the Earth orientation</td>
<td></td>
<td>±0.001</td>
</tr>
<tr>
<td></td>
<td>g) Mechanical and phase stability of DSN antennae</td>
<td></td>
<td>± &lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>h) Phase stability and clocks</td>
<td></td>
<td>± &lt; 10$^{-5}$</td>
</tr>
<tr>
<td></td>
<td>i) DSN station location</td>
<td></td>
<td>± &lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>j) Troposphere and ionosphere</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>On-board generated systematics:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) Radio beam reaction force</td>
<td>+1.10</td>
<td>±0.11</td>
</tr>
<tr>
<td></td>
<td>b) RTG heat reflected off the craft</td>
<td>−0.55</td>
<td>±0.55</td>
</tr>
<tr>
<td></td>
<td>c) Differential emissivity of the RTGs</td>
<td></td>
<td>±0.85</td>
</tr>
<tr>
<td></td>
<td>d) Non-isotropic radiative cooling of the spacecraft</td>
<td></td>
<td>±0.16</td>
</tr>
<tr>
<td></td>
<td>e) Expelled Helium produced within the RTGs</td>
<td>+0.15</td>
<td>±0.16</td>
</tr>
<tr>
<td></td>
<td>f) Gas leakage</td>
<td></td>
<td>±0.56</td>
</tr>
<tr>
<td></td>
<td>g) Variation between spacecraft determinations</td>
<td>+0.17</td>
<td>±0.17</td>
</tr>
<tr>
<td>3</td>
<td>Computational systematics:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) Numerical stability of least-squares estimation</td>
<td></td>
<td>±0.02</td>
</tr>
<tr>
<td></td>
<td>b) Accuracy of consistency/model tests</td>
<td></td>
<td>±0.13</td>
</tr>
<tr>
<td></td>
<td>c) Mismodeling of maneuvers</td>
<td></td>
<td>±0.01</td>
</tr>
<tr>
<td></td>
<td>d) Mismodeling of the solar corona</td>
<td></td>
<td>±0.02</td>
</tr>
<tr>
<td></td>
<td>e) Annual/diurnal terms</td>
<td></td>
<td>±0.32</td>
</tr>
</tbody>
</table>

Estimate of total bias/error                               +0.90                      ±1.25

\[
\alpha_p = (8.74 \pm 1.25) \times 10^{-8} \text{ cm/s}^2
\]
CHASMP Doppler Residuals For Interval III
30-Day Interval Surrounding 1996 Opposition
Example Pluto-Kuiper Express

8-year, Jupiter Gravity Assist* (JGA) trajectory to Pluto

A number of alternative trajectories have been studied for Pluto-Kuiper Express and the current favorable path for the 2004 launch date is a Jupiter Gravity Assist (JGA), arriving at Pluto/Charon 8 years later.