1. A light ray is traveling in the $+x$ direction with frequency 2000 Hz. A viewer who sees the original frame as moving with velocity in the $+z$ direction $v_z = 0.8c$ observes the light ray. What is its frequency and direction according to this viewer? You may specify the direction in terms of the tangent of the angle to the $x$ axis.

2. An electron moves in a straight line along the $+x$ direction with speed $10^5$ m/s. If there is an electric field in the $y$ direction of strength $E_y$, for what magnitude of magnetic field in what direction would this straight-line motion be maintained? Use right-hand rule conventions, and units of your choice, but explain your choices clearly.

3. A space ship accelerates uniformly in the $x$ direction (as determined in its instantaneous rest frame at each moment) with acceleration 10 m/s$^2$. In the original rest frame, this acceleration lasts for $3 \times 10^7$ s, and is followed by an equal period of deceleration. When the deceleration is completed, what are the displacement and velocity of the ship with respect to the original starting point in the original rest frame?
Midterm Exam Solutions

1. \( k' = \Lambda k \).

\( \Lambda = \begin{pmatrix}
\alpha & 0 & 0 & 0 \\
0 & \beta & 0 & 0 \\
0 & 0 & \gamma & 0 \\
0 & 0 & 0 & \delta
\end{pmatrix} \)

\( k = \begin{pmatrix}
\omega \\
0 \\
0 \\
0
\end{pmatrix} \quad (c = 2) \)

\( \omega = 2\pi \nu = 2\pi (2000 \text{ Hz}) \)

\( \omega' = \nu \omega \)

\( k_{x}' = k_{x} = \omega \)

\( k_{y}' = k_{y} = 0 \)

\( k_{z}' = \nu \omega \)

\( \nu = \frac{1}{\sqrt{1 - \nu^2}} = \frac{1}{\sqrt{1 - 0.8^2}} = \frac{1}{\sqrt{0.6}} \)

\( = \frac{1}{0.8} \)

Frequency \( \nu' = \frac{\omega'}{2\pi} = \frac{2000 \text{ Hz}}{0.8} \)

\( \rightarrow 1 \)
\[ P' = 3333 \text{ Hz} \]
\[ \tan \theta' = \frac{k'}{k_x} = \frac{\nu}{\nu'} \]
\[ = \frac{0.8}{0.6} = 1.33 \]

2. \[ F = E (\vec{E} + \vec{v} \times \vec{B}) \leq 0 \quad (c=1) \]
\[ \vec{E} \times \vec{B} = -\vec{E} \]
\[ \vec{v} \times (\vec{v} \times \vec{B}) = -\vec{v} \times \vec{E} \]
\[ = \vec{v} (\vec{v} \cdot \vec{B}) - \vec{B} \nu^2 = -\vec{v} \times \vec{E} \]
Component of \( \vec{E} \parallel \vec{v} \) arbitrary.
Set it \( \vec{v} = 0. \)
Then \[ \vec{B} = \frac{\vec{v} \times \vec{E}}{\nu^2} \]
\[ B_z = \frac{E_x}{\nu^2} \]

If \( E \) in V/m, \( B \) in T,
then \( B_z = (10^{-5}) E_y \)
If \( E \) in stat volt/cm, \( B \) in gauss,
then \( B_z = \frac{E_y}{\nu} = 3 \times 10^8 \ E_y \)

- 2 -
3. We did this in class and homework.

**Method 1)**

Constant acceleration means constant force, and force || boost is same in lab frame as in rest frame.

\[
\frac{10 m}{s^2} = \frac{d p}{dt} = \frac{d m \cdot v}{dt} = \frac{d}{dt} \left( \frac{m \cdot v}{\sqrt{1 - v^2}} \right)
\]

\[
\frac{m \cdot v}{\sqrt{1 - v^2}} = 10 \frac{m \cdot t}{s^2} = a t m
\]

\[
v = \sqrt{1 - v^2} a t
\]

\[
v^2 \left( 1 + (at)^2 \right) = (at)^2
\]

\[
v = \frac{a t}{\sqrt{1 + (at)^2}} = \frac{d}{dt} \left( \sqrt{1 + (at)^2} - 1 \right)
\]

\[
x = \frac{c^2}{a} \left( \sqrt{1 + (at)^2} - 1 \right) \quad \text{(by dim. analysis)}
\]

\[
x = \left( \frac{3 \times 10^8 \text{ m/s}}{s} \right)^2 \left( \sqrt{2} - 1 \right)
\]

\[
= \frac{10 \left( \frac{m}{s^2} \right)^2}{s^2} = 9 \times 10^{15} \text{ m}^{5/2} \text{ m}
\]

+ Deceleration \( \rightarrow 2x = 7.4 \times 10^{15} \text{ m} 
\]

\[\text{--- 3 ---}\]
The velocity is zero at the end, because for every moment with a certain positive acceleration, there is a later moment with equal negative acceleration.

Method 2)

\[
\frac{dy}{dt}_{\text{rest}} = a
\]

\[
\frac{dy}{dt}_{\text{lab}} = \frac{1}{\gamma} \frac{dy}{dt}_{\text{rest}}
\]

\[
\gamma \frac{dy}{dt}_{\text{lab}} = a \quad \gamma = \cosh y
\]

\[
\frac{d}{dt}_{\text{lab}} \sinh y = a
\]

\[
\sinh y = \sqrt{v^2 - v_f^2}
\]

\[
\frac{d}{dt}_{\text{lab}} \frac{v}{\sqrt{v^2 - v_f^2}} = a
\]

Back to method 1)
Method 2)

$$\frac{d\nu_{1ab}}{d\nu_{1ab}} = (1 - \nu^2) \frac{d\nu_{rest}}{dt}$$

$$\frac{d\nu_{1ab}}{dt} = (1 - \nu^2) \frac{3}{2} a,$$

$$\frac{d\nu}{(1 - \nu^2)^{3/2}} = a\, dt$$

$$\nu = \tanh y, \quad (1 - \nu^2) = \frac{1}{\cosh^2 y},$$

$$d\tanh y = \frac{dy}{\cosh^2 y}$$

$$\frac{d\nu}{(1 - \nu^2)^{3/2}} = \cosh y \, dy$$

$$= d\sinh y = d\left(\frac{\nu}{\sqrt{1 - \nu^2}}\right)$$

$$\frac{d}{dt} \frac{\nu}{\sqrt{1 - \nu^2}} = a$$

Back to method D.