PHY 505

Exam No. 2

Solutions

1. There is never any field inside the smaller shell \((r < R_1)\).
   (a) (i) Put a charge \(q\) on \(R_1\), no charge on \(R_2\).
   The field is evaluated by Gauss's Law.

   \[ E(r) = \begin{cases} \frac{kq}{r^2} & r > R_2 \\ \frac{kq}{R_2^2} & R_1 < r < R_2 \end{cases} \]

   The Potential \(V(r) = \int E(r) \, dr\)

   \[ V(R_2) = \frac{kq}{R_2} + C \]

   \[ V(R_1) = \frac{kq}{R_1} \left[ \frac{1}{R_1} + \frac{1}{R_2} \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \right] = C \]

   (ii) Put a charge \(q\) on \(R_2\), no charge on \(R_1\).

   \[ E(r) = \begin{cases} \frac{kq}{r^2} & r > R_2 \\ 0 & R_2 \end{cases} \]

   \[ V(R_2) = \frac{kq}{R_2} = C \]

   \[ V(R_1) = \frac{kq}{R_1} > C \]

   That is, \(C_{11} = k \left[ \frac{1}{R_1} + \frac{1}{R_2} \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \right]\)

   \[ C_{12} = C_{21} = C_{22} = \frac{k}{R_2} \]
b) \[ U = \frac{1}{2} \sum \frac{Q_i}{r_{ij}^2} \]

with \( Q_1 = 2C \)

\[ U = \frac{Q^2}{2} \left[ \frac{1}{r_1} + \frac{1}{r_2} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \right] \]

Extra Case 1:

with \( Q_1 = -\omega, \quad Q_2 = 2\omega \)

\[ \Sigma \begin{cases} \frac{KO}{r_{ij}^2} & r > R_2 \\ -\frac{KO}{r_{ij}^{3/2}} & R_1 < r < R_2 \end{cases} \]

Hence, \( U = \frac{1}{2} \left\{ \int_0^{R_1} \int_{R_1}^{R_2} \left[ \frac{KO}{r_{ij}^{3/2}} \right]^2 4\pi r^2 dr d\theta + \int_{R_1}^{\infty} \int_{R_1}^{\infty} \left[ \frac{KO}{r_{ij}^{3/2}} \right]^2 4\pi r^2 dr d\theta \right\} \]

\[ = \frac{KQ^2}{2} \left[ \frac{1}{r_1} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) + \frac{1}{r_2} \right] \]

in agreement with the result of (b).
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$$\phi_0 = \phi_0 z$$

$$\phi_0 = -\phi_0 z \text{ - electric potential}$$

We can express the potential,

$$\phi_1 = -\phi_0 \cos \theta + B_0 \sin \theta$$

$$+ \sum_{n=1}^{\infty} \left( B_n \cos(n\theta) + D_n \sin(n\theta) \right) \rho^n$$

$$\phi_2 = \phi_0 + \sum_{n=1}^{\infty} \left( A_n \cos(n\theta) + C_n \sin(n\theta) \right) \rho^n$$

(We have set an arbitrary constant equal to zero in \( B_0 \).)

Boundary Conditions

$$\phi_1(\infty) = \phi_2(\infty)$$

$$E_0 \frac{\partial \phi_1}{\partial n} \bigg|_{a} = E_0 \frac{\partial \phi_2}{\partial n} \bigg|_{b}$$

These equations hold independently for the coefficients of each homogeneous term. Except for \( \cos \theta \) terms, there are sets of 2 homogeneous equations in 2 unknowns with only the trivial zero solution.

For the \( \cos \theta \) terms:

$$-E_0 \alpha + B_1 / a = A_1 \alpha$$

$$-E_0 - B_1 / a^2 = \kappa A_1 \alpha$$

$$\alpha = \kappa \ell / a$$

or

$$A_1 \alpha^2 - B_1 = -E_0 \alpha^2$$

$$\kappa A_1 \alpha^2 + B_1 = -E_0 \alpha^2$$

Solution

$$A_1 = \frac{-2}{\kappa \ell} E_0$$

$$B_1 = \frac{\kappa \ell}{\kappa \ell - E_0 \alpha^2} E_0 \alpha^2$$
Equation for field due to electric charge on x=0 half求职 which had

\[ E_y(x) = \frac{Q}{2\pi \epsilon_0} \left( \frac{1}{x} - \frac{1}{x+h} \right) \]

\[ \phi = \frac{2}{\epsilon_0} \frac{1}{\sqrt{x}} \]

5. The Potential Change can be determined from

\[ \phi(x) = \phi(0) + \frac{Q}{\epsilon_0} \int_{0}^{x} \left( \frac{1}{r} - \frac{1}{r+h} \right) dr \]

\[ \phi(x) = \phi(0) - \frac{Q}{2\epsilon_0} \left( \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x+h}} \right) \]

\[ \phi(x) = \phi(0) - \frac{Q}{2\epsilon_0} \left( \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x+h}} \right) \]