What have we learned so far?

- Physics changed around the years 1900 to 1905 by the introduction of energy quantization by Planck and by Einstein’s realization that the speed of light must be the same in all inertial reference frames.
- Thus \( h = 4.14 \times 10^{-15} \text{ eV s} \) and \( c = 3 \times 10^8 \text{ m/s} \) became the two most important constants in nature.
- When a mass moves at \( v \sim c \) the laws must change from Newton’s mechanics to Relativity.
- When an action involves an energy and scale close to \( hc = 1240 \text{ eV nm} \) quantum effects become important.
- The concepts that explain matter and mass apply to a huge scale of lengths and energy, of which the small dimensions from nanometers (nm) to femtometer (fm) have become important to our daily life.
- Four known fundamental forces describe all interactions in nature. They differ hugely in their strength and range. They bind quarks into nuclei, nuclei into atoms, atoms into crystals and materials, and hold together the masses in the cosmos.
- At very high energies 3 of these forces approach the same strength.
The Photo Effect

• Einstein introduced $E = h \nu$ to explain the Photoeffect:

• Light knocks out electrons from material surfaces and gives them kinetic energy.

• The capability of knocking them out depends on the frequency of the light, not on the intensity.

• The energy of the light overcomes the binding energy ($WF$) of the electron:

$$K = h \nu - WF$$

• [http://lectureonline.cl.msu.edu/~mm/p/kap28/Photoeffect/](http://lectureonline.cl.msu.edu/~mm/p/kap28/Photoeffect/)

• This experiment proves that light can act as a particle.

• The binding energy $WF$ depends on the material and tells us about the energy of electrons inside crystals and surfaces.
Review: Properties of Waves

- A wave has a frequency \( \nu = \) number of oscillations per s (in Hz),
- a wave length \( \lambda = \) distance from one peak to the next (in m or nm).
- A velocity \( v = \lambda \times \nu \).
- An amplitude A.
- For sound wave \( v_s = 334 \text{ m/s} \) in air.
- When a supersonic planes velocity exceeds \( v_s \) it outruns its own sonic boom!
- A man with a flashlight in a fast car can never do that!
- Velocity of light must be the same in all inertial moving frames

- Two or more waves of the same frequency or different frequencies can be added.
- They can be diffracted and interfere with each other:
  - Young’s Double slit experiment
Special Relativity: The basics

• If light is an EM wave the laws of optics require that speed of light $c$ in free space must be the same in all inertial reference frames.
• There can be no “ether” medium that carries the light. A light wave travels through empty space!
• Otherwise a fast-moving traveler could outrun her own image!

http://hyperphysics.phy-astr.gsu.edu/hbase/relativ/star.html - c1

• This had been confirmed earlier experimentally in 1879 by Michelson.
• http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/mmexpt6.htm

• The laws of physics should be the same in all reference frames. This basic requirement will come up again and again.

• $E = h \nu$

• EM wave has a frequency $\nu$, a wavelength $\lambda$ and a speed $c$. These parameters are related:

$\lambda \nu = c$

http://hyperphysics.phy-astr.gsu.edu/hbase/ems1.html - c1
Moving clocks and time dilation

• If \( c \) is constant, then we must expect strange new physics when somebody moves at a velocity close to \( c \).
• A moving clock observed by a stationary observer, ticks more slowly when velocity is close to \( c \):

\[
    t = \frac{t'}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

• A fly lives 1 day inside a car.
• If the car moves at a velocity of

\[
    v = 0.8 \times c
\]

i.e. 80% of the speed of light the fly’s lifetime as seen by a road observer will be

\[
    t = t'/0.6 = 1.67 \text{ days}
\]

• The famous twin paradox: The twin that traveled in a spaceship at close to speed of light, ages less than the one who stays behind.

• [http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/lightclock.swf](http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/lightclock.swf)
The Amazing Atmospheric Mu Mesons

- Mu (µ) mesons are created in the upper atmosphere at \( h = 10 \text{ km} \) at a rate of \( \sim 1 \text{ per cm}^2 \text{ and sec} \).
- They live on the average in their rest frame \( t' = 2.3 \mu \text{s} \).
- They move with a speed of 0.98c
- Their travel time over 10 km is 34 \( \mu \text{s} \) and only 0.3 out 1 Million survive.
- However with time dilation their life time is \( 5 \times 2.3 = 11.5 \mu \text{s} \) and 49,000 out of a million survive.

\[ T = 300 \times 2.3 \mu \text{s} = 690 \mu \text{s} \]

Muons can be produced in particle reactions and accelerated rapidly to, say, 30 GeV.

Their time dilation factor will be

\[ \sqrt{1 - \frac{v^2}{c^2}} = \frac{E}{mc^2} = \frac{30000\text{MeV}}{100\text{MeV}} = 30c \]

Haefele and Keating Experiment in 1972 traveling around the globe.

http://hyperphysics.phy-astr.gsu.edu/hbase/particles/muonatm.html
http://hyperphysics.phy-astr.gsu.edu/hbase/relativ/airtim.html
Moving Objects and Length Contraction

• An object of length \( L' \) that moves with a speed \( v \sim c \) will be seen by a stationary observer with its dimensions in the direction of motion shortened.

\[
L = L' \sqrt{1 - \frac{v^2}{c^2}}
\]

• A car that is 5 m long at rest and travels at \( v = 0.8 \, c \) will be only

\[
L = 5m \sqrt{1 - 0.8^2} = 5m \cdot 0.6 = 0.3m
\]

• A Soccer ball will be shaped like a football with the short axis in the flight direction.

http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/lightclock.swf
How Long is the SLAC Accelerator?

• The SLAC electron linear accelerator at Stanford University is 2 miles long on the ground. How long does it appear to the electrons in the beam?
• \( L' = 2 \text{ miles} = 3,200 \text{ m} \)
• The final beam energy is 30 GeV. Thus at the half point \( E = 15 \text{ GeV} \)

\[
\sqrt{1 - \frac{v^2}{c^2}} = \frac{0.5 \text{ MeV}}{15,000 \text{ MeV}} = \frac{1}{3000}
\]

• Thus if I am riding on the electron beam through the accelerator it is only
  \( L = 3,200/3000 \sim 1 \text{ m} \) long!
• Thus it is quite easy to align the machine!
Energy and Momentum in Special Relativity

• In Newton’s mechanics every particle has a kinetic Energy \( K \) and a momentum \( p \):

\[
K = \frac{m}{2} v^2 = \frac{p^2}{2m} \\
p = m \cdot v
\]

• Because of the condition that \( c \) is the same in all frames, these rules need to be changed

• In Einstein’s mechanics every particle has kinetic energy \( K \), a momentum \( p \), and a mass energy given by \( mc^2 \):

• Kinetic energy and mass energy add up to a total energy \( E \):

\[
E = mc^2 \quad \text{Stationary particle}
\]

\[
E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{Moving particle}
\]

http://galileoandeinstein.physics.virginia.edu/lectures/mass_increase.html
Some mass energies at rest and in motion

• Rest energy of electron 511 keV
• Rest energy of the muon 106 MeV
• Rest energy of pion 140 MeV
• Rest energy of proton 938.3 MeV
• Rest energy of neutron 939.6 MeV
• Rest energy of Au nucleus 183.5 GeV
• Rest energy of U nucleus 221.6 GeV
• Rest energy of Z boson 80 GeV

• A 30-GeV electron moves with $v/c = 0.99999$. Its mass is $\sim 30$ GeV
• A 200-GeV proton moves with $v/c = 0.99987$. Its mass is $\sim 200$ GeV.
What about the Photon?

- Since photons move with the speed $c$ in any system their rest mass must be zero. You cannot stop a photon!
- Its relativistic energy is $E = h \nu$ where $\nu = \text{frequency}$. Thus high energy means high frequency.
- Wavelength and frequency are related by $\nu \times \lambda = c$

The photon momentum is

$$p = \frac{E}{c} = \frac{h}{\lambda}$$

- Example: decay of the neutral pion $\pi^0$ at rest into two photons:

  - The $\pi^0$ has a rest mass energy $E = 140 \text{ MeV}$.
  - Thus energy of each of two photons is $E_g = 70 \text{ MeV}$
  - The velocity of each photon is $c$
  - Its frequency is $\nu = 70 \text{ MeV}/h = 17 \times 10^{21} \text{ Hz}$ or $17 \times 10^{12} \text{ GHz}$
  - Its wavelength is $\lambda = c/\nu = 1.76 \times 10^{-14} \text{ m} = 17.6 \text{ fm}$, just the dimension of a nucleus
Doppler Effect and Red shift

• If a photon is emitted from a moving source in my direction, do I see any effect from the moving source?
• Yes, if the source is moving toward me, the source is “pushing” the photon in my direction. That adds energy to the photon. Since the energy of the photon is $E = h \nu$, the frequency $\nu$ increases.
• If the source, like a star, is moving away from me the photon loses energy and $\nu$ decreases.
• This is the famous Red Shift observed from receding stars and galaxies.

http://hubblesite.org/newscenter/newswire/archive/releases/2004/07
Decay of neutral mesons

• Example: decay of the neutral pi meson $\pi^0$ at rest into two photons:

• The $\pi^0$ is $E = 140$ MeV.
• Thus energy of each of two photons is $E_g = 70$ MeV
• The velocity of each photon is $c$
• Its frequency is $\nu = 70$ MeV/h = $17 \times 10^{21}$ Hz or $17 \times 10^{12}$ GHz
• Its wavelength is $\lambda = c/\nu = 1.76 \times 10^{-14}$ m = 17.6 fm, just the dimension of a nucleus
The mechanics of quantal systems

• Atoms emit light in discrete steps.
• This means electrons inside the atom must be in discrete orbitals, which cannot be explained by classical physics.
• The size of the orbits is given by the Bohr radius
  \[ R_B = 5.3 \times 10^{-11} \text{ m}, \]
• The emitted photons have energies of \( \sim 1 \text{ eV} \)
• Thus \( R_B \times E \sim 5 \times 10^{-2} \text{ nm eV} < \hbar c \) and quantum physics must be applied!

http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/hydcn.html - c1

De Broglie
The Electron as a Wave

• Einstein says: \( p = E/c = h/\lambda \) for a photon
• DeBroglie turns it around
  \[ \lambda = h/p \]
  for a particle. This trick ascribes wave properties to a particles.
• Proof: Davidson/Germer show experimentally that electron diffraction from metals is the same as that obtained from x-rays.
• However: Because of their mass electrons have a much shorter wave length than X-rays
  \[ \lambda = \frac{hc}{\sqrt{2mc^2E}} = \frac{1240eVnm}{\sqrt{2mc^2E}} \]
• For a 100 keV electron beam the wave length is
  \[ \lambda = 392 \times 10^{-5} \text{ nm} \sim 4 \text{ pm} \]
  This has led to high resolution electron microscopy.
Modern Electron Microscopy

Nano Materials Science Revealed by Advanced Electron Probes

Measuring valence electron distribution

UHV deposition & TEM lithography

Off-axis electron holography

Quantitative electron diffraction

The BNL 300kV field-emission TEM

Magnetization mapping

Atomic imaging & nano-probe EELS

Theory & image simulation

In situ magnetic imaging
Electrons as Strings

• Assume electrons are confined in the atom over a distance L
• Let’s look at it as a linear problem, a particle = a wave in a box.
• This is like a string of length L that is fixed at both ends.
• Plucking the string produces standing waves in the box, with discrete wavelengths:
  1. \( \lambda = 2L \)
  2. \( \lambda = L = 2L/2 \)
  3. \( \lambda = 2L/3 \)
     In general \( \lambda = 2L/n \)
     with \( n = 1, 2, 3 \ldots \)

• http://www.cord.edu/dept/physics/p128/lecture99_35.html

• We call \( n \) the principal quantum number of the system.
• Different values of \( n \) produce different energies inside the box:

\[
E = \frac{p^2}{2m} = \frac{h^2c^2}{2mc^2} \left(\frac{n^2}{(2L)^2}\right)
\]

• As the electron jump form a higher \( n \) to a lower \( n \) it looses energy which is given off as discrete light quanta.
Electrons in atoms & quantization of L

- Electrons are bound into atoms by the attraction of the negative charge of the electron by positive charge of the nucleus.
- If the electron circles around the nucleus on a radius R with velocity v, it acts like a spinning top. The spinning motion is expressed as an angular momentum L
  \[ L = mvR = \frac{h}{\lambda} R, \]
- Again has standing waves around the circle 2πR
  \[ L = n_1 \frac{h}{2\pi} \]
- \( N_1 = l = 0, 1, 2, 3 \ldots \)

- Like a spinning top the atom can wobble, so the direction of the spinning axis can change but the spinning motion itself is constant.
- The intensity of the spinning motion is quantized like
  \[ L = \sqrt{l(l+1)} \cdot \frac{h}{2\pi} \]
Wave functions and Tunneling


- The standing wave inside a potential through is called the wave function of the particle, $\Psi$. Its square gives the probability of finding the particle in a particular place in space.

- If the walls of the trough are not very high, the wave function can leak out through the wall.

- It has tunneled through the barrier even though it does not have enough energy to climb over it.

- Electron tunneling is the basis for most transistors and has huge practical applications.

Erwin Schroedinger invented the wave function, which made quantum mechanics a powerful theory.
Second Homework Set, due Feb. 10, 2005

1. What is the evidence that photons can be treated like particles?
2. What is the evidence that electrons can be treated like waves?
3. You have heard of the Twin Paradox. Do you really believe it?
4. If you see a sleek sports car driving by on the road at a speed of 0.99c, would it look stunted, elongated or unchanged to you? Explain!
5. Electrons that are bound in an atom have discrete, i.e. quantized orbitals. What property of the electron is responsible for that?
6. How can the electrons bound in an atom emit light with discrete wavelength?
How to submit Homework

• You have 3 possibilities:

1. Submit it to me in class on the date it is due.
2. Put it in the TA’s (Xiao Shen) mailbox in the Physics Department main office on or before the due date
3. Submit it him by e-mail at the address: xshen@ic.sunysb.edu

• Please DO NOT submit it to me by e-mail