PHY313 - CEI544
The Mystery of Matter
From Quarks to the Cosmos
Spring 2005

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www.physics.sunysb.edu PHY313
What have we learned last time?

- We completed understanding the consequences of particle-wave duality: *Heisenberg’s Uncertainty Relations*: I cannot know location *and* motion of a particle exactly, simultaneously; I cannot know the energy and the timing of an event exactly, simultaneously.

\[ \Delta p \cdot \Delta x \approx \hbar; \Delta E \cdot \Delta t \approx \hbar \]

- Based on these very basic principles we derived that a particle trapped inside a box cannot sit still but must constantly move. The size of the box and the mass of the particle determined the kinetic energy range. In the atom the energy scale is \( \sim \) tens of eV; in the much smaller nucleus the energy scale of nucleon motion is \( \sim \) MeV.

- We learned that the Pauli Principle governs how electrons are fitted into atomic states and nucleons into nuclear states. No two particles can have all the same quantum numbers.

- Because electrons and nucleons have intrinsic spin, with two orientations (up and down) we can put two electrons into the lowest atomic orbit, and 4 nucleons (2 protons and 2 neutrons) into the lowest orbit of the nucleus.

- The decay of particles and nuclei is statistical: At any time the same fraction of the still present particles or nuclei decays. After one half-life \( T_{1/2} \) half of the particles/nuclei has decayed. After ten half lives the original activity has decayed by a factor \( \sim1000 \).
The Nucleus: The hard core of the atom

- **Rutherford Scattering** of 4-MeV alpha particles proved around 1912 that the nucleus was a dense, hard object, not a “pudding”.
- The proof was the observation that the α’s were scattered into the backward direction, rather than penetrate through the “pudding”.
- The scattering force was the repulsive Coulomb interaction between the positive nucleus and the positive nucleus.
- From the probability of scattering α-particles backwards, the size of the nucleus could be estimated.

- Lord Rutherford, head of Cavendish Laboratory

- A table top experiment:
The scattering Cross Section

http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/rutsca2.html
http://www.waowen.screaming.net/revision/nuclear/rsanim.htm

• The “cross section” is proportional to the area \( \pi R^2 \) of the scattering object. It depends on the energy of the scattering \( \alpha \)-particle.

• Cross sections are expressed in units of \( 10^{-24} \text{ cm}^2 \), which is called “a barn (bn)”.

• The cross section for scattering of 10 MeV \( \alpha \)’s from Uranium is 12 bn = 1.2 x \( 10^{-24} \text{ cm}^2 \).

• Thus the radius is about less than 12 fm.
Hard scattering as a general method

- Observation of events at angles > 90 degrees indicates hard scattering center.
- The search for hard scattering centers is a general method to search for hidden substructure.
- In the 1970’s such scattering measurements using much higher energies lead to the discovery of quarks inside the nucleons.
- Above 27-MeV α-particles one observes already deviations from a solid nucleus. This indicates that many alpha’s are now penetrating the nucleus.
Nuclear constituents

- The nucleus of an atom has a charge number Z and a mass number A. e.g. the Helium nucleus has Z=2 and A=4.
- It was first thought that the nucleus consists of A protons and A-Z electrons. This was soon ruled out.
- In fact the nucleus consists of Z protons and A-Z neutrons, e.g. Helium4 has 2 protons and 2 neutrons, lead 208 has 82 protons and 208-82= 126 neutrons
- The nomenclature is \(_2\text{He}^4\) and \(_{82}\text{Pb}^{208}\)
- These nucleons are packed together as tightly as possible, as indicated by the increase in size as nucleons are added.

\[ R(A) = 1.2 \, \text{fm} \cdot 3\sqrt{A} \]

\[ \text{Volume} \approx R^3 \approx A \]

- The volume of the nucleus grows linearly with A, which means that the nucleus looks like a drop of water that grows linearly as water molecules are added. This indicates that the force between nucleons must be short range.
- For \(_2\text{He}^4\) \( R = 1.2 \times 1.587 = 1.9 \, \text{fm} \)
- For \(_{82}\text{Pb}^{208}\) \( R = 1.2 \times 5.92 = 7.1 \, \text{fm} \)
The masses of nuclei

- The mass of a nucleus is made up from the masses of its constituent nucleons plus the attractive strong interaction between all the nucleons minus the energy of the Coulomb repulsion between the protons.
- It becomes easier to add uncharged neutrons in heavier nuclei, rather than positively charged protons.
- These rules form the “Valley of Stability” of stable nuclei.
- Soon after Pb no stable nuclei exist because of very strong Coulomb repulsion.
- Adding more neutrons to a nucleus with a given Z results in “isotopes”. They are mostly radioactive.
The Binding Energies of Nuclei

- The binding energy of all nucleons inside the nucleus is given by
  \[ B(A,Z) = Z M_h + (A-Z) M_n - M(A,Z) \]
- B/A gives the binding energy per nucleon, i.e. the average energy that must be provided to free up a nucleon from inside the nucleus.
- The binding energy/nucleon starts very low in light nuclei, with some spikes at some "magic numbers".
- The largest binding with 8.8 MeV occurs in Fe. For higher nuclei the BE decreases again because of the increasing Coulomb repulsion.

http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/nucbin.html

- The nuclei He\(^4\), Be\(^8\), O\(^{16}\), Ca\(^{40}\), Pb\(^{208}\) are especially strongly bound.
Energy release in fission reactions

- Because of the increasing Coulomb repulsion of the assembled protons nuclei above Pb are increasingly ready to fission into two much lighter nuclei.
- This process frees up energy because the fission products are more tightly bound than the parent nucleus.
- For example, $^{235}\text{U}$ fissions when hit by a neutron

$$^{235}\text{U} + n \rightarrow ^{144}\text{Ba} + ^{89}\text{Kr} + 3n$$

- The energy release per atom is

$$Q = \{M(^{235}\text{U}) - M(^{144}\text{Ba}) - M(^{89}\text{Kr}) - 2m_n\} = 173\text{MeV}$$

- 1 kg of $^{235}\text{U}$ has $25 \times 10^{23}$ atoms
- So 1 kg of U releases $173 \times 25 \times 10^{23}$ MeV
- This is $Q = 70 \times 10^{12}$ J = $20 \times 10^6$ kW hrs, equivalent to the electricity consumption of $\sim 2$ Million homes for 1 hour or 17,000 tons of TNT.
- 1 kg of Uranium can satisfy the energy needs of 176 US citizen for 1 year.

← Note that all Z and A numbers left and right add up.
Energy Release in Fusion Reactions

http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/fusion.html

- In the sun 4 protons combine to form \( ^4\text{He} \) which is very tightly bound.
- This releases 26 MeV per 4 protons
- 1 kg of hydrogen molecules has \( 3 \times 10^{26} \) molecules which can produce \( 40 \times 10^{26} \text{ MeV} = 1.7 \times 10^8 \text{ kW hrs} \)
- This reaction fuels the sun.
- In actual fact one uses the reaction \( \text{D} + \text{T} \rightarrow ^4\text{He} + \text{n} + 17.6 \text{ MeV} \)
  Each kg of this reaction satisfies the needs of 676 US citizen for 1 year.

- The fusion of deuterium contained in sea water could satisfy the energy needs of mankind for the foreseeable future. Plasma fusion tries to achieve the goal of producing a fusion reactor. (ITER)
Hydrogen burning: Net result:

\[ 4p \rightarrow \text{He}^4 + 25 \text{ MeV Energy} \]

Nuclear reactions for hydrogen burning in the sun
The force between nucleons

- The nuclear force is not known at a fundamental level, but well understood at a working level.
- By and large the strong forces between protons and neutrons are the same.
- The force consists of a “long range” (~1 fm) attraction, and a “short range” repulsion (otherwise nucleons would fall into each other!)
- The “nuclear potential” in which all nucleons move is produced by the combined interaction of all nucleons. It is a bit different between neutrons and protons because of the Coulomb repulsion for protons.
The Yukawa force

- In 1935 Yukawa recognized a deep new principle about forces:
  
  \textit{Forces are mediated by the exchange of particles}

- Like football players interact by exchanging the football, nucleons interact by exchanging $\pi$ mesons ($mc^2 = 140\text{MeV}$)

- Heisenberg provides the $\pi$ mesons!

- His relation allows to produce a mass energy of 140 MeV for a time

\[ \Delta E \cdot \Delta t \approx \hbar \]

\[ \Delta T = \hbar / 140\text{MeV} = 4 \cdot 10^{-24} \text{s} \]

- During this time the p can move a distance of 1.2 fm, which is just right. Then it fades away.
Nuclear Structure

- Nucleons move around in levels of excitations, just as electrons in atoms do. However, since their combined interaction produces the potential in which they are moving, their motions can strongly deform the nucleus.

- **Superdeformation**, in which the nucleus has the shape of an American football with a 2:1 axis ratio, is a striking deformation effect.

- It is strikingly observed in the gamma transitions between spinning motions of the nucleus. With each emission of a photon (γ-ray) the nucleus slows down its rotation by 2 units of angular momentum $2\hbar$. 

![Diagram of nuclear structure and transitions](image-url)
Chart of the Nuclei

“Magic” proton numbers
2, 8, 20, 28, 50, 82

“Magic” neutron numbers
N = Z

Stable Nuclei

Known radioactive Nuclei
Nuclear Isotopes

- Nuclei at the edges of the valley of stability contain extra protons or neutrons. They can be stable or unstable and are of great practical importance.
- The stable ones are used as tracers.
- Unstable isotopes are used for Positron Emission Tomography (PET), for radiodating, for medical tracer elements, or even for medical irradiation.
- $^{16}$O has 8 protons and 8 neutrons
- The lighter isotopes have fewer neutrons than protons: $^{14}$O has 8 p, 6 n
- The heavier ones have more neutrons: $^{22}$O has 8 p, 14 n

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Abund.</th>
<th>Lifetime</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{14}$O</td>
<td></td>
<td>70.6 s</td>
<td>0</td>
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<tr>
<td>$^{15}$O</td>
<td></td>
<td>122.2 s</td>
<td>1/2</td>
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<tr>
<td>$^{16}$O</td>
<td>99.8%</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$^{17}$O</td>
<td>0.04%</td>
<td></td>
<td>5/2</td>
</tr>
<tr>
<td>$^{18}$O</td>
<td>0.2%</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$^{19}$O</td>
<td></td>
<td>26.9 s</td>
<td>5/2</td>
</tr>
<tr>
<td>$^{20}$O</td>
<td></td>
<td>13.5 s</td>
<td>0</td>
</tr>
<tr>
<td>$^{21}$O</td>
<td></td>
<td>3.4 s</td>
<td></td>
</tr>
<tr>
<td>$^{22}$O</td>
<td></td>
<td>2.2 s</td>
<td></td>
</tr>
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</table>
Radio-dating

- [http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/cardat.html](http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/cardat.html)

- The lifetimes of radioisotopes are independent of chemistry, temperature etc.

- Because of the long half life of $^{14}$C The ratio of $^{14}$C/$^{12}$C can be used to date a piece of wood or anything that contains carbon.

- The initial equilibrium ratio is $^{14}$C/$^{12}$C = $1.3 \times 10^{-12}$

  After 1 half life (5692 yrs) this ratio has changed to $0.6 \times 10^{-12}$ etc.

  Trees as old as 5000 yrs are easily measured.

- $^{11}$C is an important PET isotope.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Abund.</th>
<th>$T_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^9$C</td>
<td></td>
<td>0.127 s</td>
</tr>
<tr>
<td>$^{10}$C</td>
<td></td>
<td>19.4 s</td>
</tr>
<tr>
<td>$^{11}$C</td>
<td></td>
<td>20.4 min</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>98.9%</td>
<td></td>
</tr>
<tr>
<td>$^{13}$C</td>
<td>1.1%</td>
<td></td>
</tr>
<tr>
<td>$^{14}$C</td>
<td></td>
<td>5692 yrs</td>
</tr>
<tr>
<td>$^{15}$C</td>
<td></td>
<td>122 s</td>
</tr>
<tr>
<td>$^{16}$C</td>
<td></td>
<td>0.74 s</td>
</tr>
</tbody>
</table>
What is the age of the earth?

- We need a radioisotope with a half life of ~ billion years = $10^9$ yrs
  $^{238}\text{U} T_{1/2} = 4.49 \times 10^9$ yrs $\rightarrow 206\text{Pb}$
  $^{235}\text{U} T_{1/2} = 7.1 \times 10^8$ yrs $\rightarrow 207\text{Pb}$
  $^{204}\text{Pb} T_{1/2} = 1.4 \times 10^{17}$ yrs
- The experiment involves measuring the abundance ratios of $^{207}\text{Pb}/^{204}\text{Pb}$ and $^{206}\text{Pb}/^{204}\text{Pb}$
- Regardless of what the initial ratio of the Uranium isotopes was, these data points must fall on a straight line to be consistent
  - The age of the earth comes out to be $4.55 \times 10^9$ years.

A recent article on the longest dating times:
Radioisotopes in Medicine

- Nuclear isotopes used for diagnostic, palliation and treatment.
- Radioisotopes in Medicine mostly produced with commercial cyclotrons.
- Technicium 99m discovered at BNL in 1970’s used today in 85% of all diagnostic nuclear medicine, 20 Million doses annually.
- Allows tracing of blood movement through heart etc: 200,000 procedures each year.
- 36,000 nuclear medicine procedures each day!
- Tin 117m most promising agent against bone pain and bone metastases.
- Radio-Isotopes neat to be produced again and again because they decay.
How do we make new elements here on earth?

Chart of the Nuclei

Heavy Ion Fusion

18O projectile

Fr\textsuperscript{210} product

Gold Target

18O

He

N

Ca

Ni

Sn

Pb

Fr\textsuperscript{215}

neutrons

How do we make new elements here on earth?

Heavy Ion Fusion
Glenn Seaborg

1912-1999

Discovered Plutonium: The first new element produced in large amounts, T1/2 = 24,390 yrs

Sg 106

Was the only living person for whom an element was named.
History of the Synthesis of Heavy Elements

1898
Polonium (Z = 84), Radium (Z = 88)
1899
Actinium (Z = 89)

Discovery of radioactivity
by A. H. Becquerel

1908
Radon (Z = 86)

1917
Protactinium (Z = 91)

1934
Enrico Fermi proposes to irradiate
Uranium with neutrons to
synthesize heavier elements

1939
Francium (Z = 87)

1896

1941
Plutonium (Z = 94)

1940
Astat (Z = 85), Neptunium (Z = 93)

1944
Americium (Z = 95), Curium (Z = 96)

1949
Berkeley (Z = 97)

1950
Californium (Z = 98)

1940

USA

1952

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Hot Fusion – Irradiation of Heavy Target Nuclei with Light Ions

Cold (»Gentle«) Fusion – Irradiation of Pb and Bi Targets

UNILAC ACCELERATOR AT GSI

(P. Armbruster, »Spektrum der Wissenschaft« (german edition of the »Scientific American«), december 1996)
The search for super-heavy nuclei

Chart of the Nuclei

Predicted Island of Stability

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Radioactivity: beta decay

- Nuclei outside of the valley of stability undergo radioactive decay.
- The simplest decay is that of the neutron
  \[ n^0 \rightarrow p^+ + e^- + \bar{\nu} + 780\text{keV} \]
- This decay conserves electric charge.
- Beta decay is mediated by the weak interaction and therefore very slow. The available energy is shared by all the secondary particles.
- If the neutron is inside a nucleus the beta decay can also happen:
  \[ ^{14}_6\text{C} \rightarrow ^{14}_7\text{N} + e^- + \bar{\nu} \]
- However, when inside a nucleus a proton can also decay into a neutron:
  \[ ^{11}_6\text{C} \rightarrow ^{11}_5\text{B} + e^+ + \nu \]
- This is a widely used decay because the positron \( e^+ \) that is emitted decays immediately into 2 \( \gamma \)-rays emitted in opposite directions.
Neutron reactions

- Neutron scattering
  
  \[ n + {}^{12}C \rightarrow n + {}^{12}C \]
  
  \[ n + {}^{12}C \rightarrow n' + {}^{12}C + 4.4\text{MeV} \gamma \]

- Neutron capture shows distinct resonances where the n is captured into excited nuclear states.

  \[ n + {}^{235}U \rightarrow {}^{236}U \rightarrow \text{fission} \]
Boron Neutron Cancer Therapy

Once a promising treatment for distributed brain cancers (Gliomas) which cannot be treated any other way. Despite promising early results not accepted by clinical community.

The Boron Neutron Capture (BNC) Reaction
- The $^{10}$B-atomic nucleus is unique among the light elements since it has a great propensity to capture slow neutrons. The $^{11}$B-atomic nucleus does not capture slow neutrons in this manner.
- About 2.4 MeV of kinetic energy is released to propel the $^7$Li$^{+}$ and the $^4$He$^{2+}$ ions which are produced by this fission reaction.
- The neutron was discovered by Chadwick in 1932, and the $^{10}$B (n, α) $^7$Li reaction (or BNC) was characterized by Taylor in 1934.

Selective Cell Killing with Boron Neutron Capture Therapy (BNCT)
- By themselves, both slow neutrons and $^8$B-atoms are harmless. Together, they combine to give a lethal cell-killing nuclear reaction.
- The $^8$He$^{2+}$ and $^7$Li$^{+}$ ions formed in the BNC reaction only travel about one cell diameter in tissues and produce ionization along their trajectories.

The BNCT Cell-Killing Mechanism
- Nucleus (DNA)
- Cytoplasm
- Cell membrane
- Slow neutron
- Photon
- $^8$He$^{2+}$
- $^7$Li$^{+}$
Nuclear fission

- Heavy nuclei from Thorium onwards are very unstable and are easily induced to fission by bombardment with neutrons, releasing about 200 MeV.

\[ n + ^{235}\text{U} \rightarrow ^{236}\text{U} \rightarrow \text{fission} + 2n \]

- This reaction has a huge cross section of 708 bn for slow neutrons

- \(^{238}\text{U}\) fissions very poorly with slow neutrons, but it "breeds" Plutonium

\[ ^{238}\text{U} + n \rightarrow ^{239}\text{U} + e^- \rightarrow ^{239}\text{Np} + e^- \rightarrow ^{239}\text{Pu} \]

- \(^{239}\text{Pu}\) is excellent fission material

http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/fission.html

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Abund.</th>
<th>n/fission</th>
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<tbody>
<tr>
<td>233U</td>
<td>0.1%</td>
<td>2.51</td>
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<tr>
<td>235U</td>
<td>0.72%</td>
<td>2.47</td>
</tr>
<tr>
<td>238U</td>
<td>99.27%</td>
<td>22.91</td>
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</table>
Fourth Homework Set, due Feb. 24, 2005

1. Why is it reasonable to consider the proton and the neutron as members of a close family, and what is the name for this family of two?

2. What was the important experimental observation that the nucleus was not a soup but a solid rock of matter, and where was this experiment done?

3. Which nuclei are most tightly bound? Why the heaviest nuclei prone to fission?

4. Energy can be produced through fission of the heaviest nuclei or fusion of the lightest. Which ones produce more energy per kilogram of matter, and why?

5. Why is 14C such a good nucleus to be used for radioactive dating?

6. How can we produce new nuclei and new elements in the laboratory?