SUPERGRAVITY DUALS

OF $N=1$ SUPERSYMMETRIC

GAUGE THEORIES

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FROM ITS VERY INCEPTION, SUPERGRAVITY WAS CLEARLY A VERY DEEP AND RICH SUBJECT. 25 YEARS AGO IT WAS HARD TO ANTICIPATE HOWEVER, THAT IT CONTAINS LARGE N GAUGE THEORY. CONFORMAL GAUGE THEORIES IN 3+1 DIMENSIONS ARE DESCRIBED BY THE \( \text{AdS}_5 \times X_5 \) SOLUTIONS OF TYPE IIB SUGRA. (I WILL EMPHASIZE A SPECIFIC EXAMPLE \( X_5 = T^{1,1} = \frac{\text{SU}(2) \times \text{SU}(2)}{\text{U}(1)} \) ) Romans

Breaking of conformal invariance produces a variety of gauge theory phenomena: logarithmic running of couplings, chiral symmetry breaking, confinement, etc. ALL SEEN FROM SUGRA.

In \( N=1 \) SUSY \( \text{SU}(N) \) gauge theory XSB is true.
the appearance of gluino condensate

\[ \langle \Omega \Omega \rangle = \Lambda^3 e^{2\pi i n/M}, \quad n=1, \ldots, M \]

It breaks the \( \mathbb{Z}_{2M} \) chiral symmetry down to \( \mathbb{Z}_2 \); \( n \) labels the \( M \) inequivalent vacua.

Recently, this pattern of chiral symmetry breaking was observed geometrically, in extensions of the AdS/CFT correspondence to non-conformal \( N=1 \) SUSY gauge theories.

I.K., Strassler; Maldacena, Nunez; Vafa; ...

\[ S^2 \quad S^3 \]

\[ SU(N+M) \times SU(N) \] gauge theory lives on \( N \) D3-branes and \( M \) wrapped D5-branes on the CONIFOLD.
The Conifold is a 6-dimensional cone defined by the equation
\[ \sum_{i=1}^{4} z_i^2 = 0 \]
for 4 complex variables.

The metric created by the D-branes at the apex is a WARPED PRODUCT:
\[ ds_{10}^2 = h^{-\frac{1}{2}}(r)(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + h^{\frac{1}{2}}(r) \, ds_6^2 \]
I.R., A. Tseytlin

\[ ds_6^2 = \text{explicitly known metric of the cone;} \]
\[ r = \text{the radius of the cone; \( T^3 \) is its base.} \]

The CHIRAL SYMMETRY acts geometrically as \[ z_i \rightarrow z_i e^{i\alpha} \]
\[ \alpha = \frac{\pi n}{M} \; ; \quad n = 1, 2, \ldots, 2M. \]

However, the warp factor has a zero:
\[ h(r^*) = 0 \Rightarrow \text{NAKED SINGULARITY!} \]
The Einstein metric on $T''$ is
\[\begin{align*}
\frac{\ell^2}{g} ds^2_{T''} &= \frac{1}{g} \left( dy + \cos \theta_1 \, d\varphi_1 + \cos \theta_2 \, d\varphi_2 \right)^2 + \\
&\quad + \frac{1}{6} \sum_{i=1}^{2} \left( d\theta_i^2 + \sin^2 \theta_i \, d\varphi_i^2 \right)
\end{align*}\]

\[\varphi, \theta \in [0, 2\pi) \; ; \; \varphi_1, \varphi_2 \in [0, 2\pi) \; ; \; \theta_1, \theta_2 \in [0, \pi)\]

The harmonic 2- and 3-forms are
\[\omega_2 = \frac{1}{2} \left( \sin \theta_1 \, d\theta_1 \wedge d\varphi_1 - \sin \theta_2 \, d\theta_2 \wedge d\varphi_2 \right),\]
\[\omega_3 = e^u \wedge \omega_2,\]
\[e^u = dy + \cos \theta_1 \, d\varphi_1 + \cos \theta_2 \, d\varphi_2\]
\[\int \omega_2 = 4\pi \; ; \; \int \omega_3 = 8\pi^2,\]

To have $M$ units of RR 3-form flux, we need
\[\frac{1}{4\pi^2} \int F_3 = M;\]

The solution with this extra flux through $T''$ is

I.K., Tseytlin
\[ F_3 = \frac{d^1}{2} N w_3 \quad B_2 = \frac{d^1}{2} 3 g_5 M \ln(\frac{r}{r_0}) w_2 \]

The dilaton \( \Phi = 0 \).

\[ ds^2 = h^{-\frac{1}{2}}(r) dx \cdot dx + h^{\frac{1}{2}}(r) (dr^2 + r^2 ds^2_{T^11}) \]

\[ h(r) = \frac{\sqrt{g_5}}{r^4} \left( N_0 + \frac{3}{2 \pi} g_5 M^2 \ln(\frac{r}{r_0}) + \frac{1}{4 \pi} \right) \frac{27}{16} \]

The 5-form field strength \( \widetilde{F}_5 = F_5 + * F_5 \), \( F_5 \sim N(r) \, \text{vol}(T^11) \).

\[ N(r) = N_0 + \frac{3}{2 \pi} g_5 M^2 \ln(\frac{r}{r_0}) \]

The number of colors \( N \) in \( SU(N) \times SU(N+M) \) has become scale dependent.

As \( \ln(\frac{r}{r_0}) \) decreases by \( \frac{2\pi}{3g_5 M} \),

\[ N(r) \rightarrow N(r) - M \]

This is the cascade of Seiberg dualities.
Connection with the NSVZ $\beta$-functions.

\[
\frac{d}{d\ln(N)} \frac{8\pi^2}{g_1^2} \sim 3(N+N)-2N(1-\gamma)
\]

\[
\frac{d}{d\ln(N)} \frac{8\pi^2}{g_2^2} \sim 3N-2(N+N)(1-\gamma)
\]

$\gamma$ is the anomalous dimension of $Tr(A_iB_j)$

\[
\gamma = -\frac{1}{2} + \mathcal{O}\left(\frac{N^2}{M^2}\right).
\]

\[
\frac{8\pi^2}{g_1^2} - \frac{8\pi^2}{g_2^2} \sim 6M \ln\left(\frac{\Lambda}{\mu}\right)
\]

In $\text{SUGRA}$ we have

\[
SB_2 = (2\pi i) 3g_s M \ln\left(\frac{\Lambda}{\mu_0}\right)
\]

Using the relation

\[
\frac{1}{2\pi i g_s} \int SB_2 = \frac{4\pi^2}{g_1^2} - \frac{4\pi^2}{g_2^2}
\]

we find exact agreement with FT.

The factor 3 in the $\text{SUGRA}$ calculation is geometrical; related to self-duality of $G_3$. The correct $\beta$-function also follows from $W \sim G_3^{1/2}$ Vafa.
This singularity is a signal of the chiral symmetry breaking.

To achieve a non-singular solution of the supergravity equations of motion, the conifold must be deformed to

\[ \sum_{i=1}^{\tilde{N}} z_i^2 = \epsilon^2, \quad \text{I.K., Strassler} \]

This breaks the \( \mathbb{Z}_{2\tilde{N}} \) symmetry down to \( \mathbb{Z}_2 \): \( z_i \rightarrow -z_i \).

\[ ds_{10}^2 = h^{-\frac{1}{2}}(r) (-dt^2 + dx^2) + h^{\frac{1}{2}}(r) d\tilde{s}_6^2. \]

Now the warp factor \( h(r) \) has NO unwanted zeroes.

\[ \sqrt{h(0)} \sim g_{ym}^2 M = \text{'t Hooft coupling of the SU}(M) gauge theory found in the infrared.} \]
A fundamental string at the apex \((r=0)\) along the world volume directions \(\mathbb{R}^{3,1}\) has tension \(\sim \frac{1}{\sqrt{h(0)}}\).

It is finite \(\Rightarrow\) CONFINEMENT

In contrast, for \(\text{AdS}_5 \times S^5\), \(h = \frac{L^4}{r^4}\) blows up at \(r=0 \Rightarrow\) NO CONFINEMENT FOR \(N=4\) SUPER-YM.

**IN THE \(N=1\) SU(M) GAUGE THEORY**

**THERE ARE CONFINING 9-STRINGS**

Quarks \(\leftrightarrow\) anti-quarks

They connect 9 probe quarks with 9 corresponding anti-quarks.

Wilson loops in the fully antisymmetric representation with 9 indices.
The ten vectors $T_q, \ q=1,2,\ldots, M-1$.

$T_q = T_{M-q}$ ( $q \rightarrow M-q$ exchanges quarks with antiquarks).

$T_M = 0$ ( $M$ quarks form a colorless object, the baryon).

In our SUGRA dual the $q$-string is described by $q$ coincident fundamental strings at the apex.

The blown-up 3-sphere has metric (60.953)

$$ds_3^2 = b \ g_5 \ M \alpha' [d\gamma^2 + \sin^2 \gamma (d\theta^2 + \sin^2 \theta \ d\phi^2)]$$

and $M$ units of RR 3-form flux $F_3 = 2 \ M \alpha' \ sin^2 \gamma \ sin \ \theta \ d\gamma \ d\theta \ d\phi$.

$\gamma \in [0, \pi]$ is the azimuthal angle.
The R-R flux blows up the 9 F-string into a 03-brane wrapped over an $S^2$ at fixed azimuthal angle $\gamma$. C. Herzog, I.K.

$$T \sim b^2 \sin^2 \gamma + \left( \gamma - \frac{\sin 2\gamma}{2} - \frac{9}{M} \right)^2$$

Minimising with $\gamma$, we find

$$\gamma - \frac{8\gamma}{M} = \frac{1-b^2}{2} \sin(2\gamma)$$

If $b=1$, then $\gamma = \frac{8\gamma}{M}$

$$T_q \sim \sin(\frac{8\gamma}{M})$$

This formula is valid in softly broken $N=2$ and in MBGD.

For the $K3$ soltu, $b \approx 0.933 \Rightarrow T_q$ is almost the same.

For the Maldacena-Nunez SUGRA dual, which also has a blown-up $S^3$, $b = 1$ exactly.
q-string tensions
The D-brane calculation has the exact $q\to M\to q$ symmetry crucial for the gauge theory interpretation.

Our calculation is reliable for large $g_s M$. To describe pure $N=1$ SU($N$) gauge theory we need small $g_s M$.

The KS background describes a different gauge theory: SU($N+M$) \times SU($N$) which "cascades" to pure SU($N$) only far in the IR.

A lot remains to be understood...

Nevertheless,

\[
\frac{T_q}{T_{q'}} \approx \frac{\sin(\pi q/M)}{\sin(\pi q'/M)}
\]

may be a good model for what to expect from the LATTICE (Lucini + Teper).

Del Debbio et al.